# Managing Warranty Inventory for Multi-Generational High-Tech Devices

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# Motivation

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GARMIN
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Layup
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Layup
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Dogleg
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APPROACH
#11 Par 4
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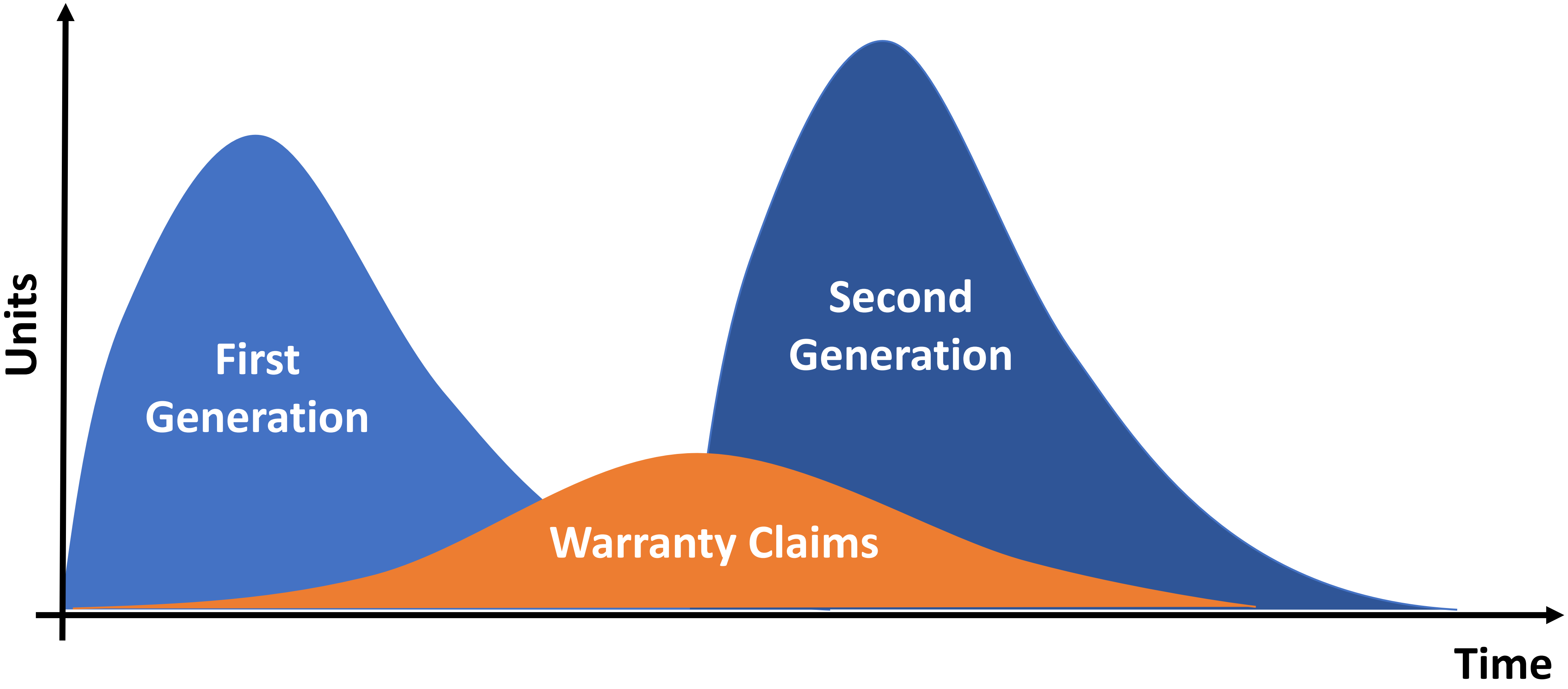
# When Should We Stop Production?



# The Big Questions

**Timing**

When should we stop production?

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**Timing** — When should we stop production?

**Quantity** — How much should we order when we stop?

# Outline

Literature Review → Model → Simulations → Future Work

# Literature Review

- Commonly known as:
  - Last Time Buy (LTB)

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  - Lifetime Buy
  - End of Life Buy
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- Motivated by spare parts setting
  - Supplier has discontinued an essential component and manufacturer must make LTB

# Literature Review

Table 1: Supply Options Considered in Addition to the Last Time Buy

| Paper | Repair | Harvest Parts from Returns | Additional Production | Product Trade-Ins |
|---|---|---|---|---|
| Moore (1971) | | | | |
| Ritchie and Wilcox (1977) | | | | |
| Fortuin (1980) | | | | |
| Fortuin (1981) | | | | |
| Teunter and Haneveld (1998) | | | | |
| Teunter and Fortuin (1999) | | ✓ | | |
| Teunter and Haneveld (2002) | | | ✓ | |
| Cattani and Souza (2003) | | | | |
| Kleber and Inderfurth (2007) | | ✓ | | |
| Inderfurth and Mukherjee (2008) | | ✓ | ✓ | |
| Bradley et al. (2009) | | | | |
| van Kooten and Tan (2009) | ✓ | | | |
| Leifker et al. (2012) | | | ✓ | |
| Pourakbar and Dekker (2012) | | | | |
| Pourakbar et al. (2012) | ✓ | | | |
| Inderfurth et al. (2013) | | ✓ | ✓ | |
| van der Heijden and Iskandar (2013) | ✓ | | | |
| Pourakbar et al. (2014) | | ✓ | | ✓ |
| Behfard et al. (2015) | ✓ | | | |
| Cole et al. (2015) | | | | ✓ |
| Cole et al. (2016) | | | | ✓ |

# Assumptions

- We consider only devices that are too costly to repair
- Zero lead time
- Until the final period, warranty claims are satisfied as they arrive
- Leftover units have no salvage value

# Notation

## Parameters

- $T$ - number of periods
- $c_p$ - production cost per unit
- $c_s$ - shortage cost per unit
- $c_f$ - fixed operational production cost per period
- $c_h$ - holding cost per unit per period

## Decision Variables

- $t$ - time of final order or final period of production
- $q$ - final order quantity

# Notation

## Demand Distributions

- $D_i$ - random variable representing demand in period $i$ where $i = 1...T$
- $f_i^j$ - pdf of cumulative demand from period $i$ to period $j$
- $F_i^j$ - cdf of cumulative demand from period $i$ to period $j$

# Expected Cost

$$\min_{t,q} \quad \underbrace{c_f t}_{\text{Operational Costs}}$$

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$$\min_{t,q} \quad \underbrace{c_f t}_{\text{Operational Costs}} + \underbrace{c_p \sum_{i=1}^{t} \mathbb{E}[D_i] + c_p q}_{\text{Production Costs}}$$

# Expected Cost

$$\min_{t,q} \quad \underbrace{c_f t}_{\text{Operational Costs}} + \underbrace{c_p \sum_{i=1}^{t} \mathbb{E}[D_i] + c_p q}_{\text{Production Costs}} + \underbrace{c_h \sum_{i=t+1}^{T} \mathbb{E}\left[\left(q - \sum_{j=t+1}^{i} D_j\right)^+\right]}_{\text{Holding Costs}}$$

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# Solution Properties

When the warranty demand is:

1. Independent period to period
2. From a family of infinitely-divisible distributions (e.g. Normal)
3. Non-negative in each period

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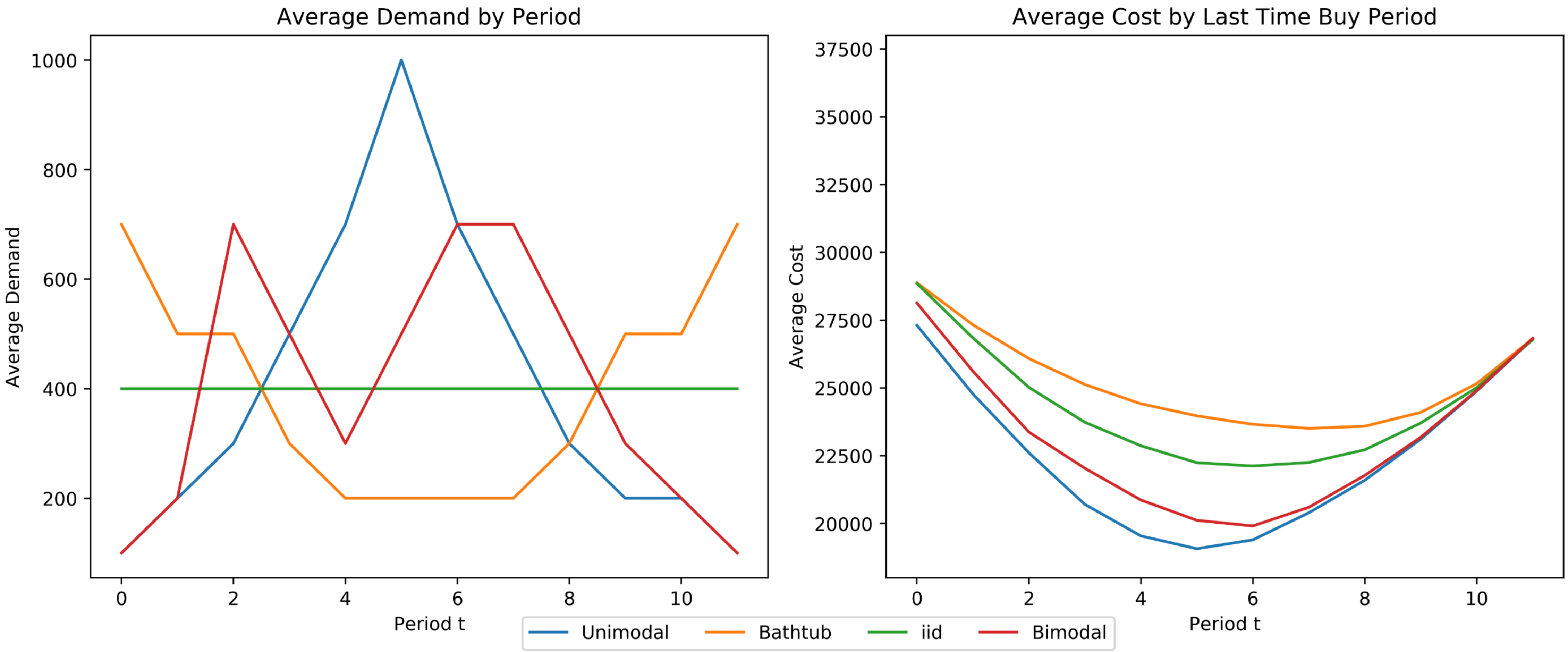
The Expected Cost is convex in $q$ for a given $t$

$q^*(t)$ is non-increasing in $t$

# Simulation Results



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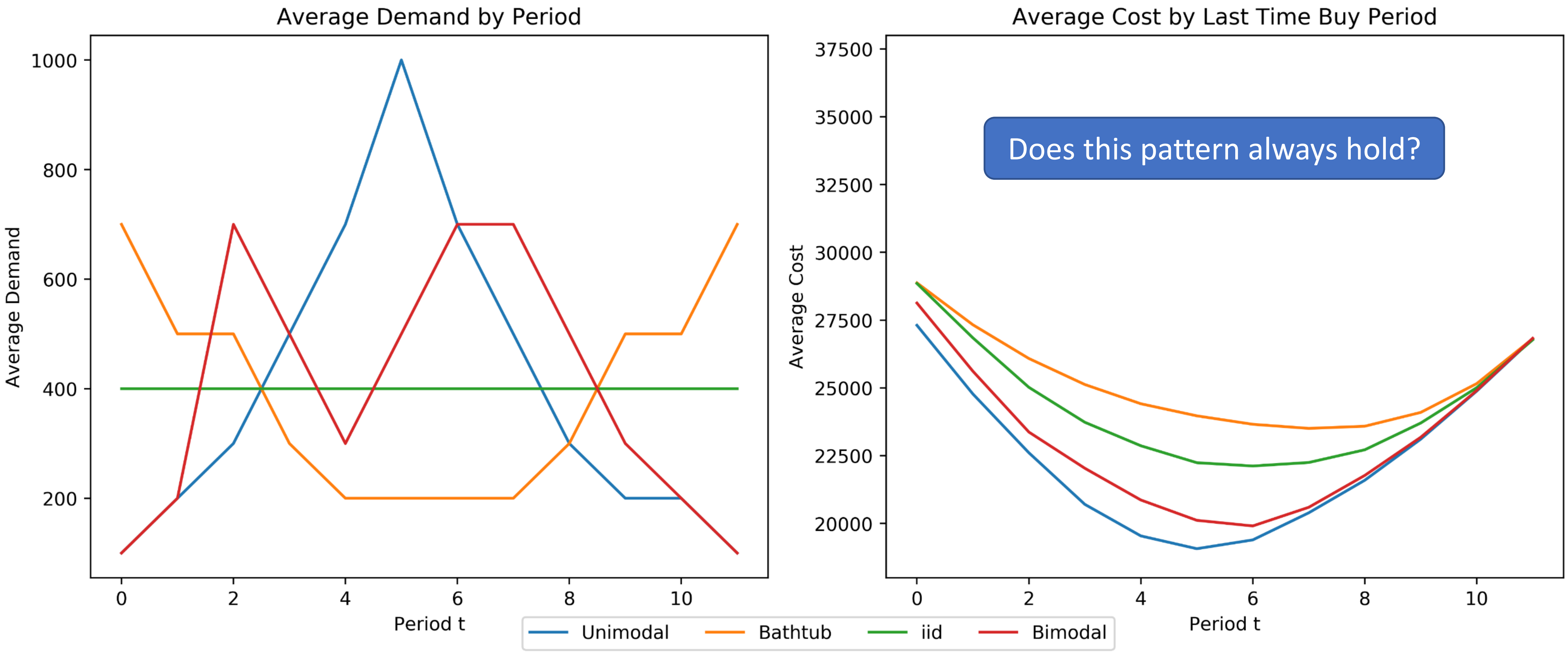

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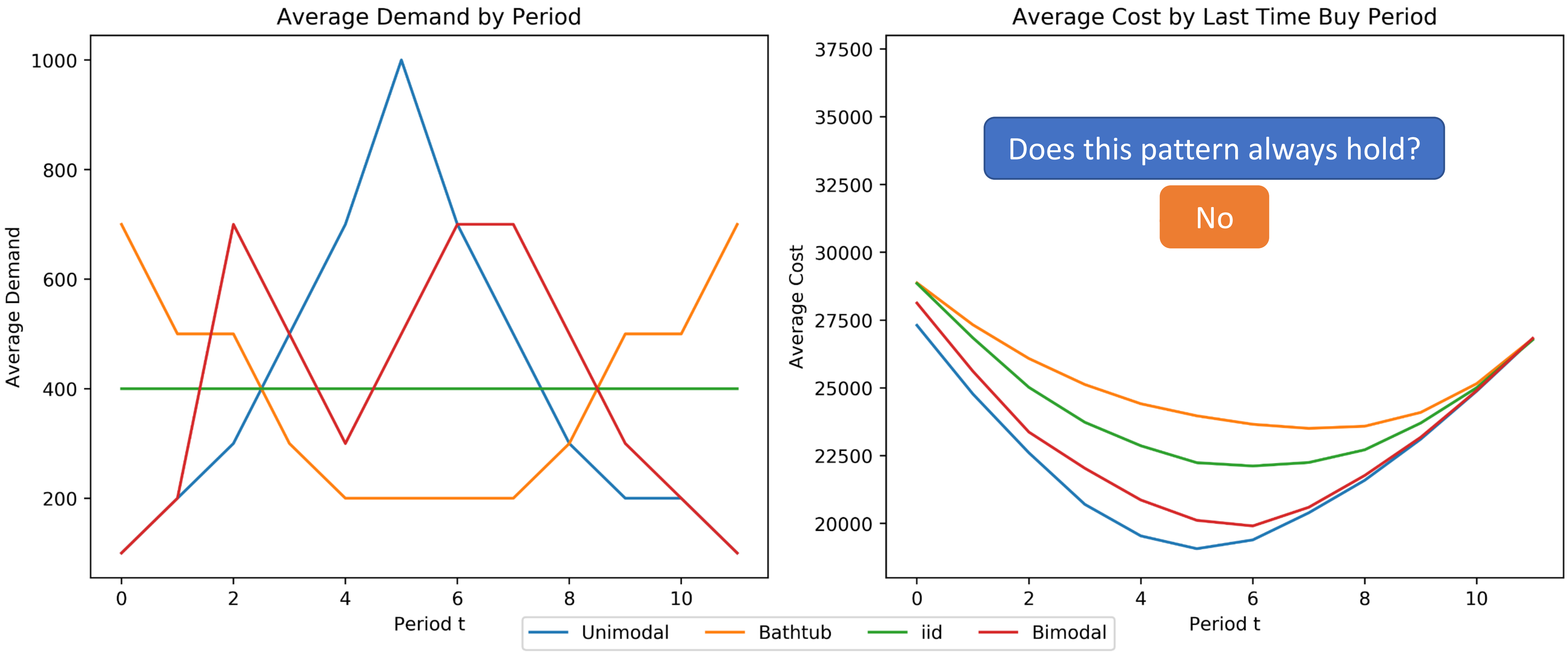
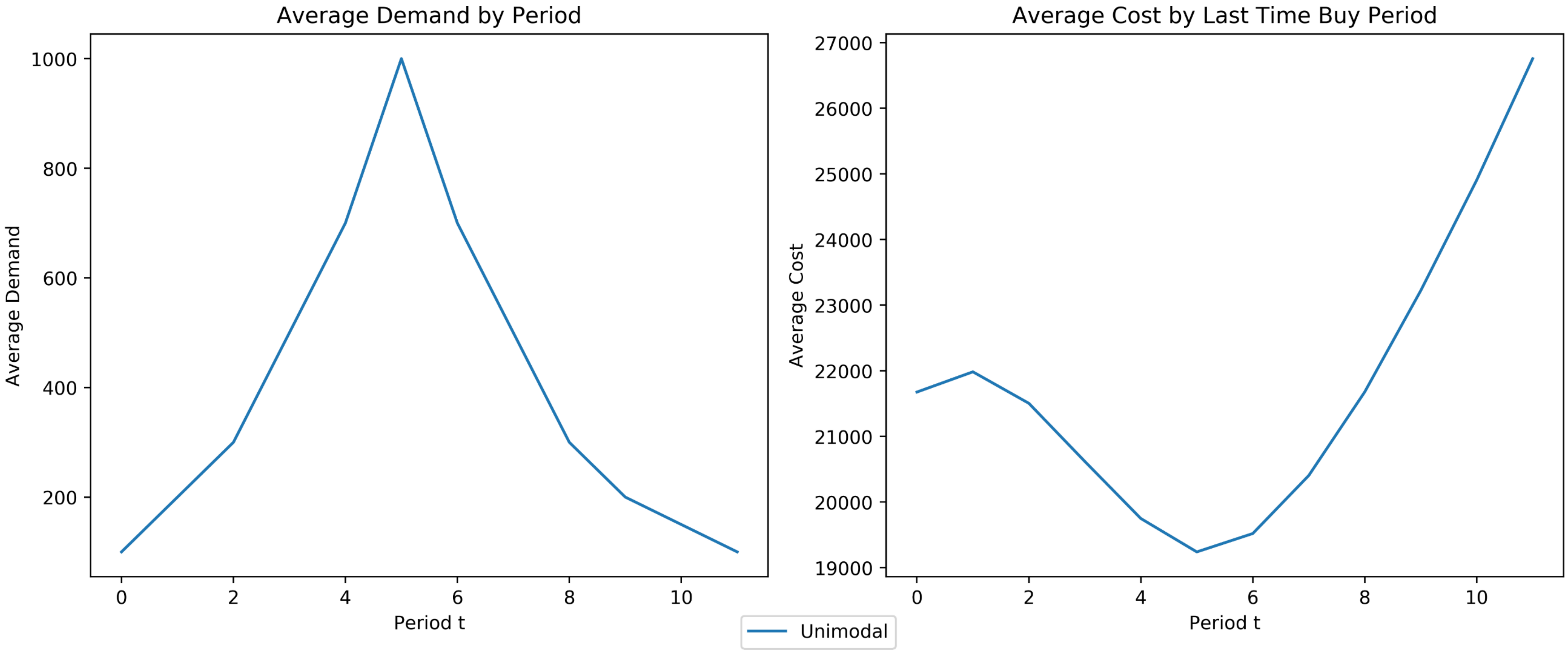


# Counterexample



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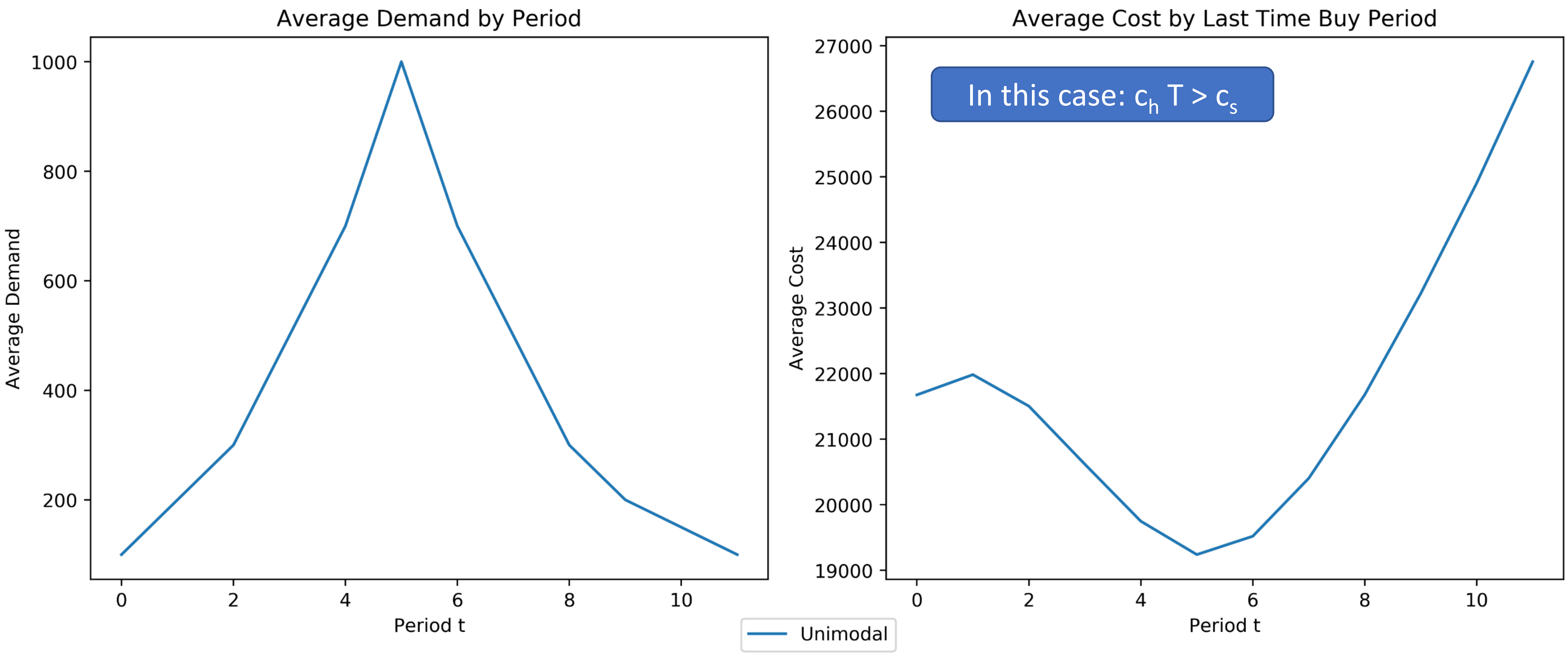

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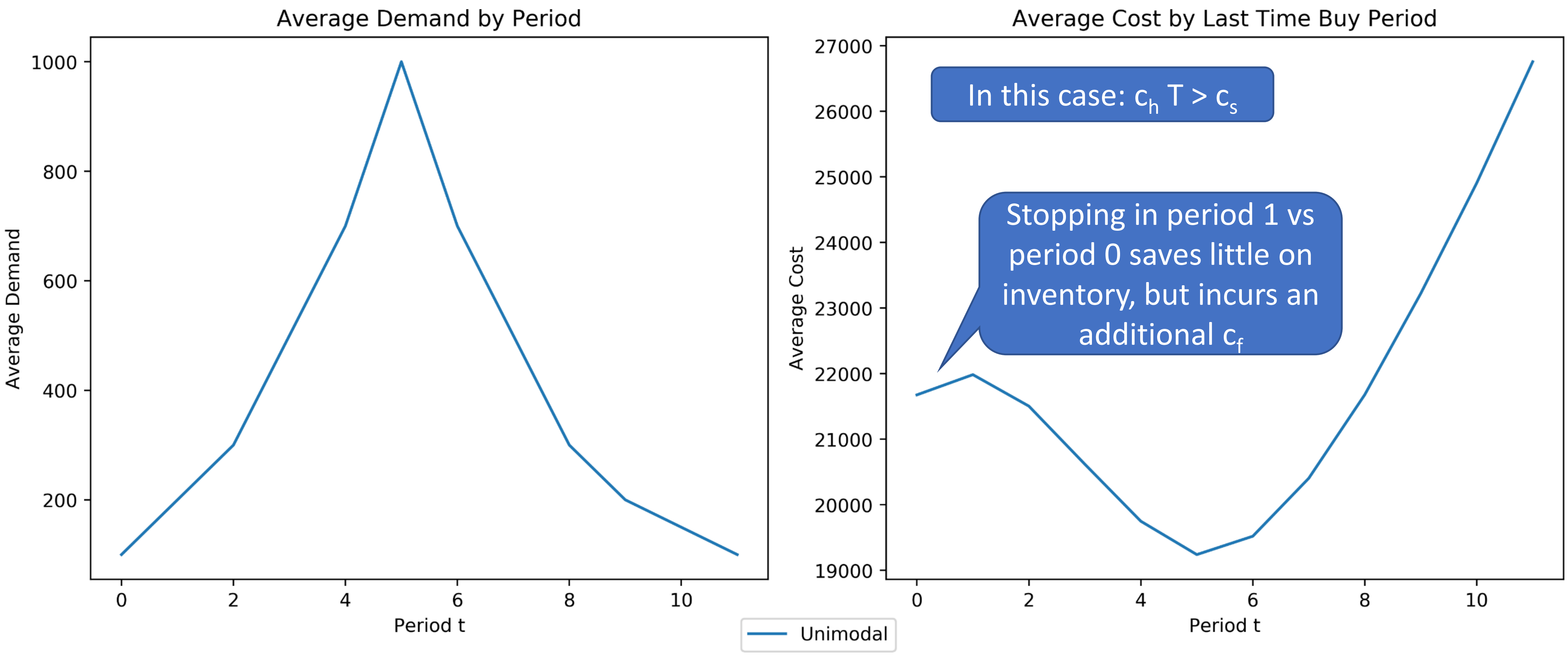

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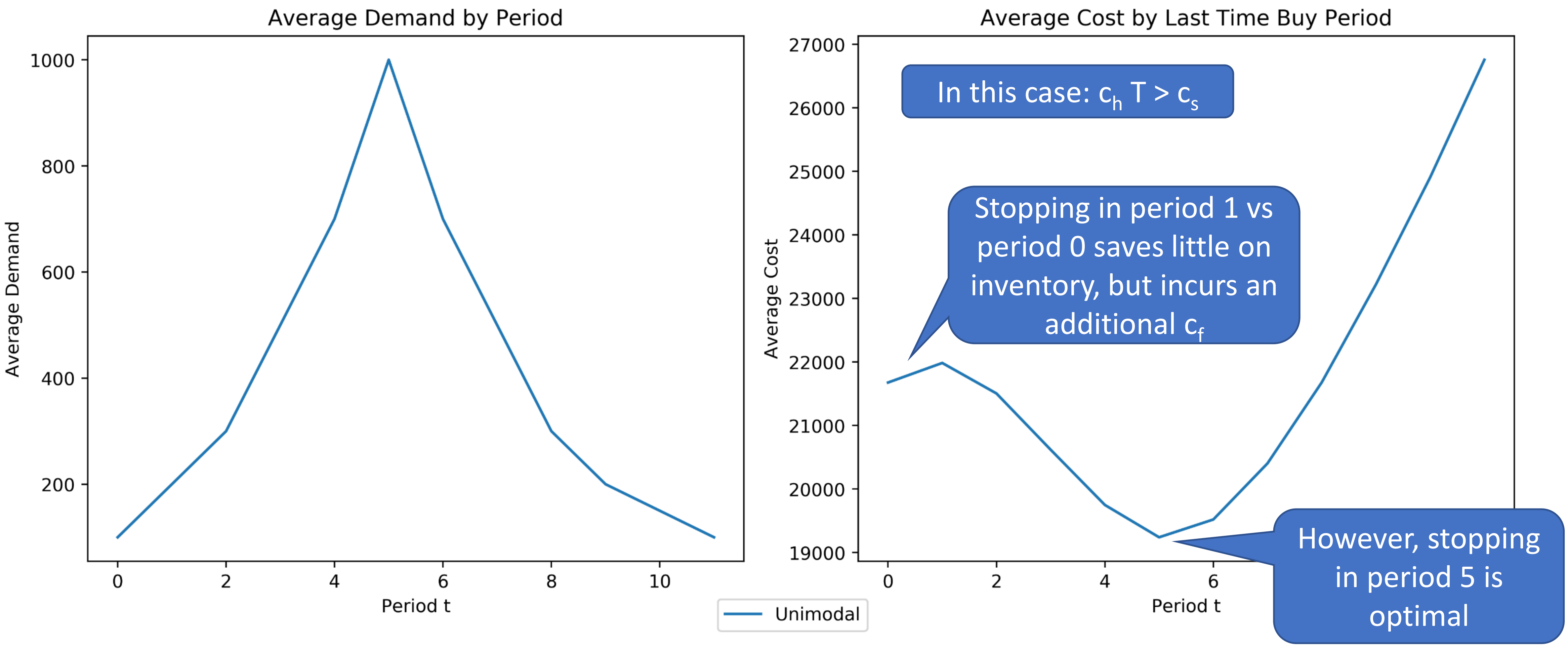

# Revisiting the Demand Assumptions

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To learn about the true device failure rate

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# New Demand Model

- Given a population of devices of varying ages
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# New Demand Model

- Given a population of devices of varying ages
- Failures occur according to an increasing failure rate model

Let $n_t(j)$ be the number of devices in period $t$ that are $j$ periods old

Let $\hat{h}_t(j)$ be the hazard rate for devices that are $j$ periods old estimated in period $t$

Then the demand in any given period $t$ can be expressed as:

$$D_t = \sum_{j=1}^{T} \text{Bin}(n_t(j), \hat{h}_t(j))$$

# Stochastic Dynamic Program with Learning

In each period,

1) Update failure rate estimates by age
2) Calculate expected cost of:
   a) Making the Last Time Buy now
   b) Best option involving producing now and making the LTB later

If the DP value function were convex, we might be able to solve the problem, but we already showed that convexity is not guaranteed

# Stochastic Dynamic Program with Learning

## Decision Variables

- $y_t$ is set to 1 if we produce in period $t$, 0 otherwise
- $q_t$ is the amount we produce in period $t$

## State Variables

- $I_t$ is the inventory at the end of period $t$
- $\hat{h}_t(j)$ is the estimate of the hazard rate updated in period $t$ for devices that are $j$ periods old
- $n_t(j)$ is the number of devices in period $t$ that are $j$ periods old

# Stochastic Dynamic Program with Learning

Assume that we are currently in period $t$:

$$G_t(y_t, q_t; I_{t-1}, y_{t-1}, \vec{\hat{h}}_{t-1}, \vec{n}_{t-1})$$

# Stochastic Dynamic Program with Learning

Assume that we are currently in period $t$:

$$G_t(y_t, q_t; I_{t-1}, y_{t-1}, \vec{\hat{h}}_{t-1}, \vec{n}_{t-1}) = \min_{\substack{y_t : y_t \leq y_{t-1} \\ q_t : q_t \leq M y_t}} c_f y_t + c_p q_t + \mathbb{E}\left[c_h(I_{t-1} + q_t - D_t)^+ + c_s(D_t - I_{t-1} - q_t)^+\right]$$

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Current Period: Operational and Production Costs, Holding Costs, Shortage Costs

# Stochastic Dynamic Program with Learning

Assume that we are currently in period $t$:

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$$= \min_{\substack{y_t : y_t \leq y_{t-1} \\ q_t : q_t \leq M y_t}} \quad c_f y_t + c_p q_t + \mathbb{E}\left[c_h (I_{t-1} + q_t - D_t)^+ + c_s (D_t - I_{t-1} - q_t)^+\right]$$

$$+ \underbrace{\mathbb{E}\left[G^*_{t+1}((I_{t-1} + q_t - D_t)^+, y_t, \vec{\hat{h}}_t, \vec{N}_t)\right]}_{\text{Expected Cost to Go}}$$

# Motivating Example

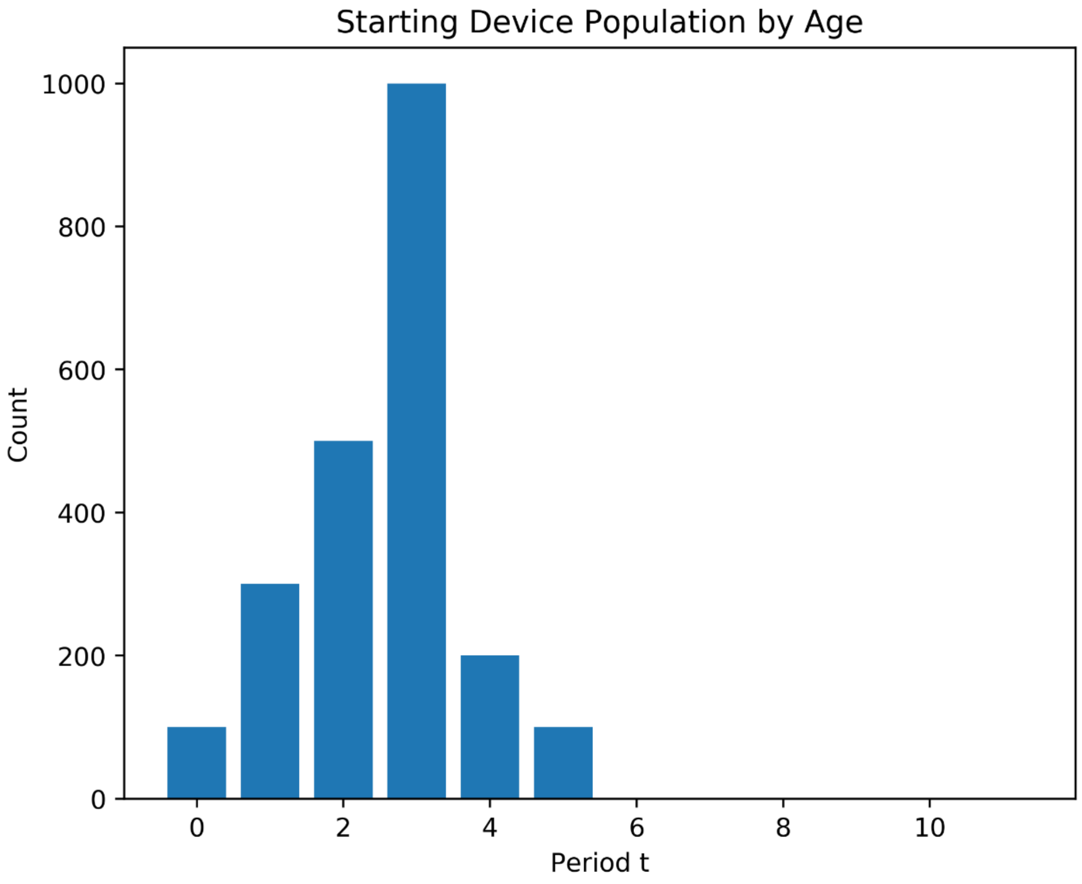
Two possible hazard rates, but incomplete information due to an immature population of devices

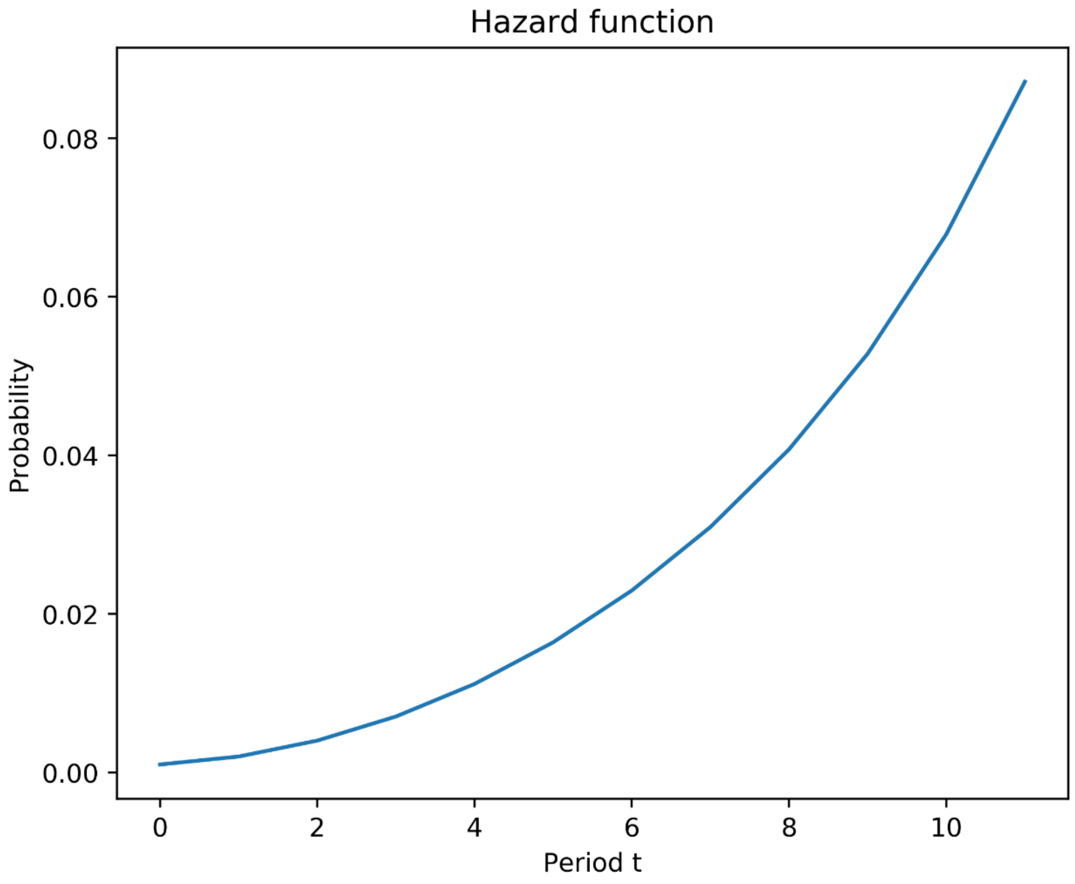
Assumptions

- No learning in these examples
- The warranty period is 12 months
- Replaced devices are no longer eligible for warranty claims
- In each period, demand is observed before being satisfied

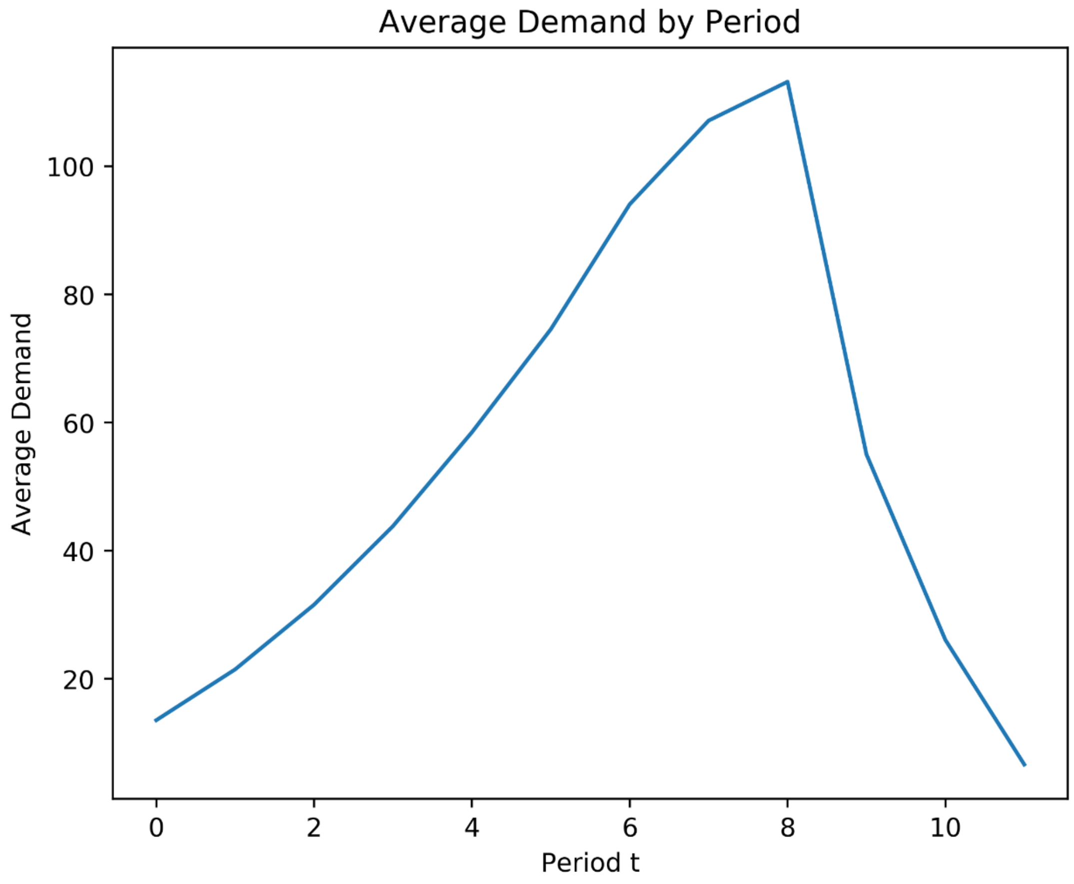
Purpose: To show the value of delaying the Last Time Buy

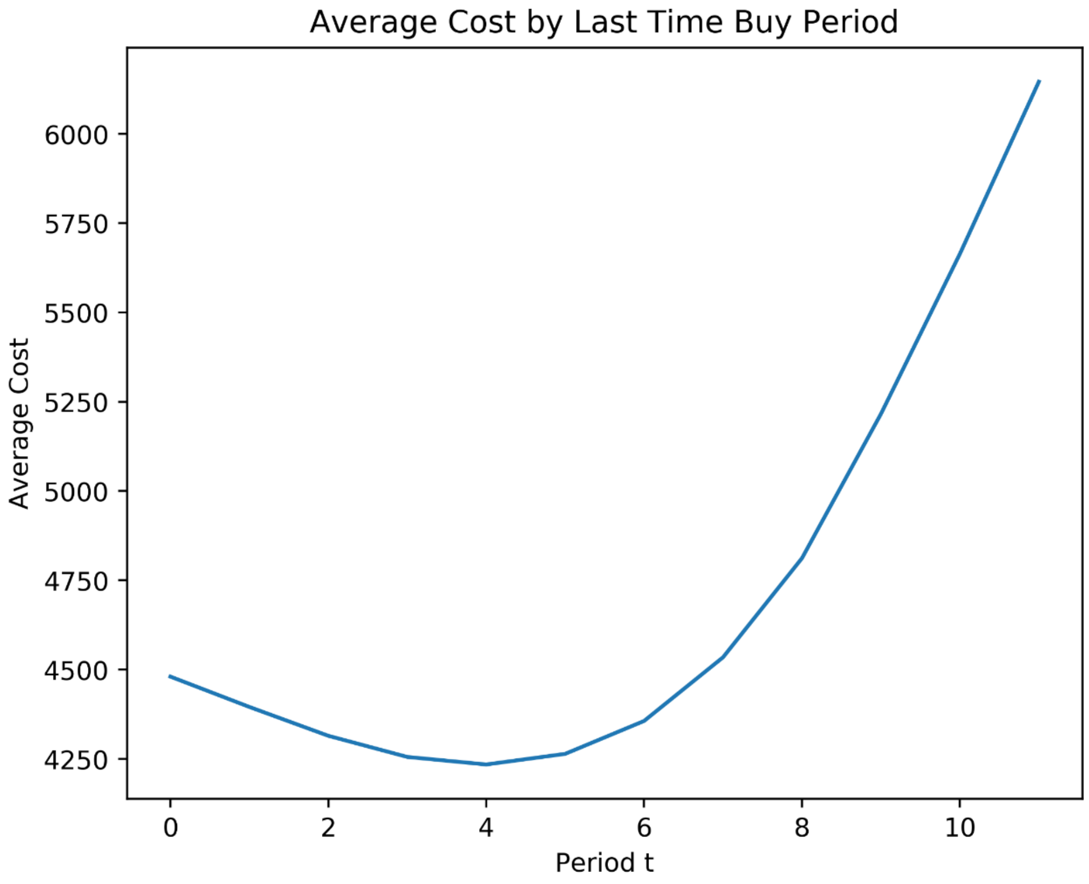
# Potential Warranty Claims



# Resulting Demand and Cost



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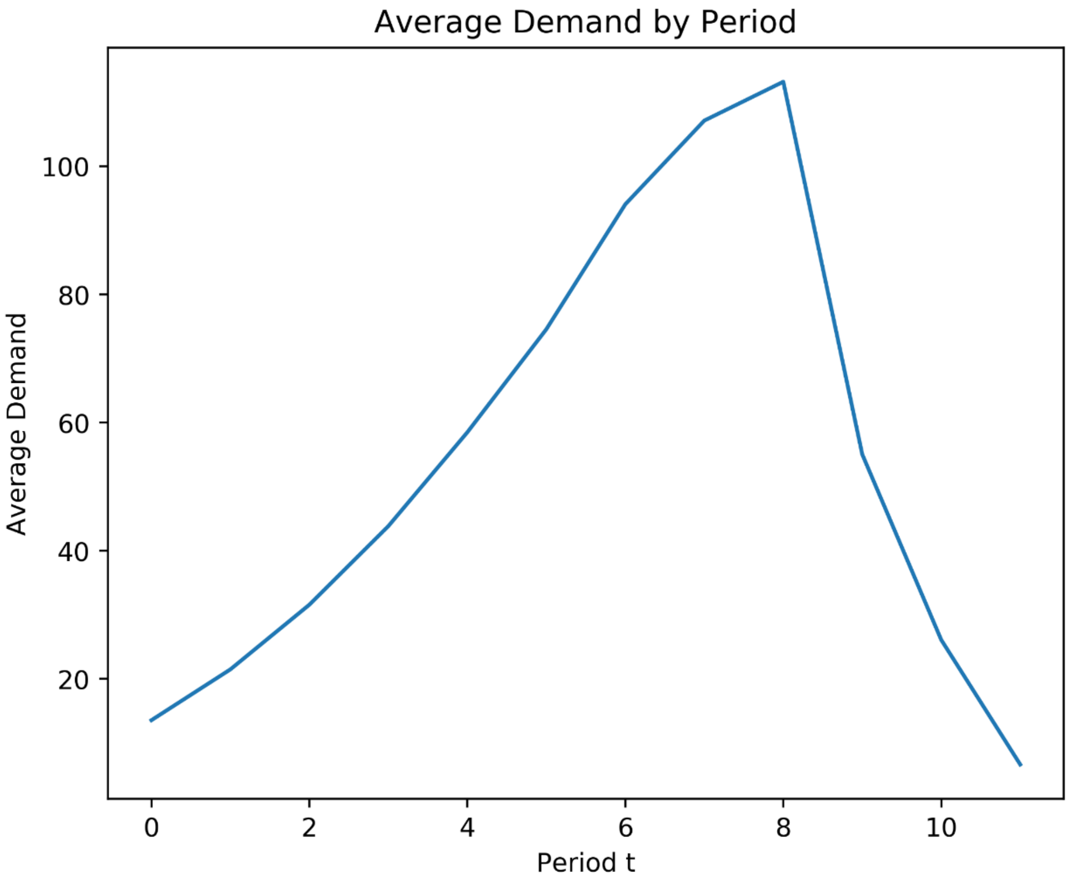
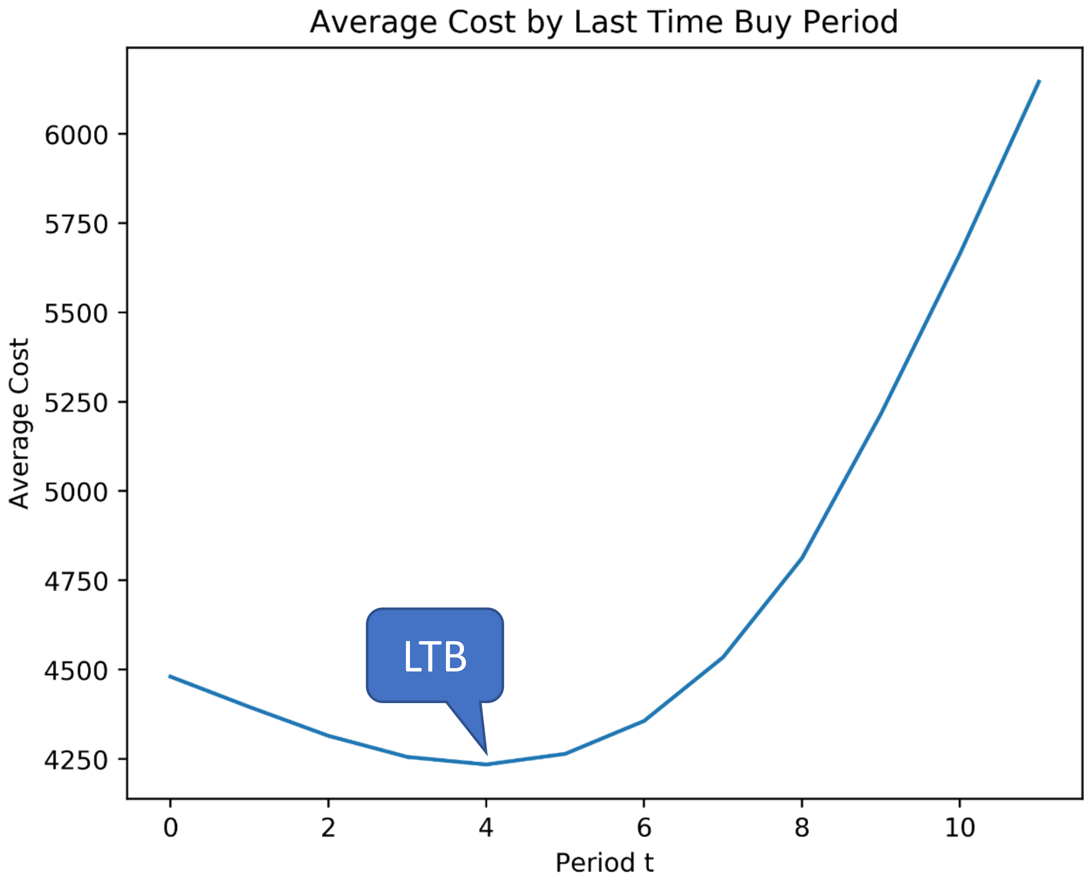
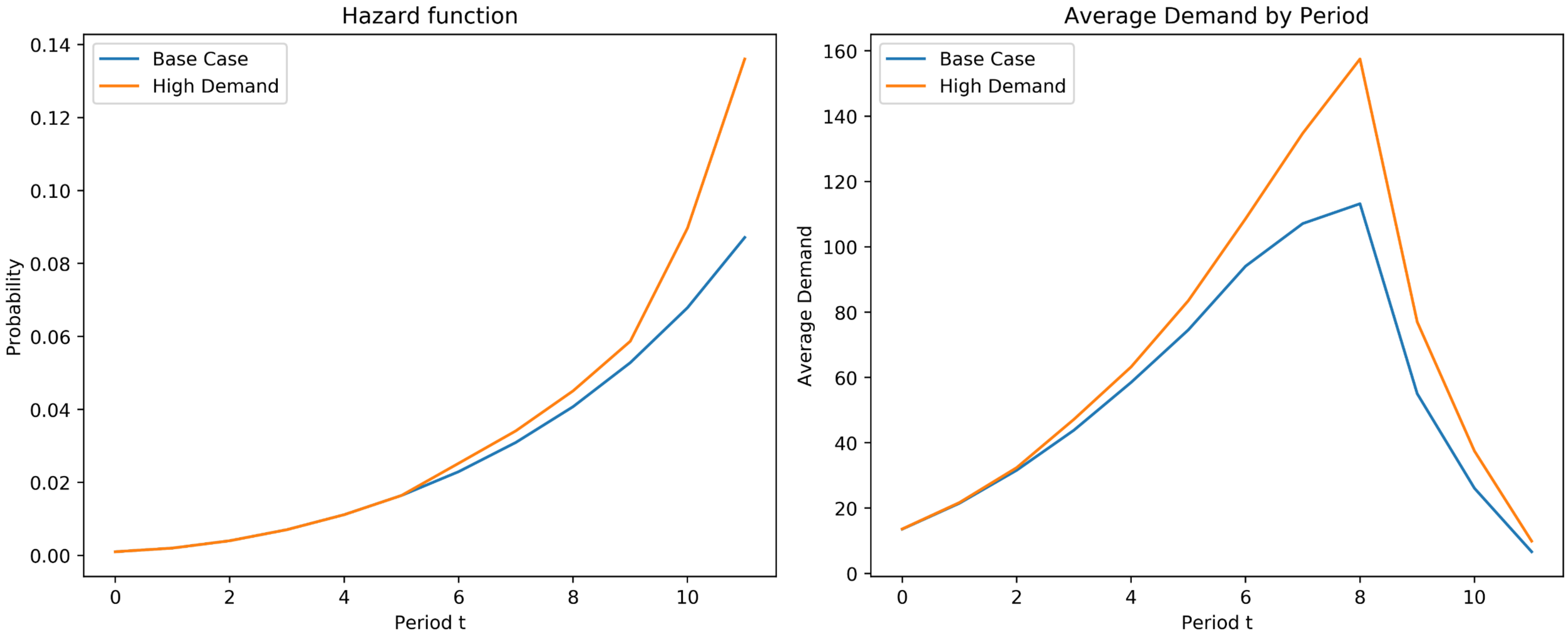


# Two Scenarios



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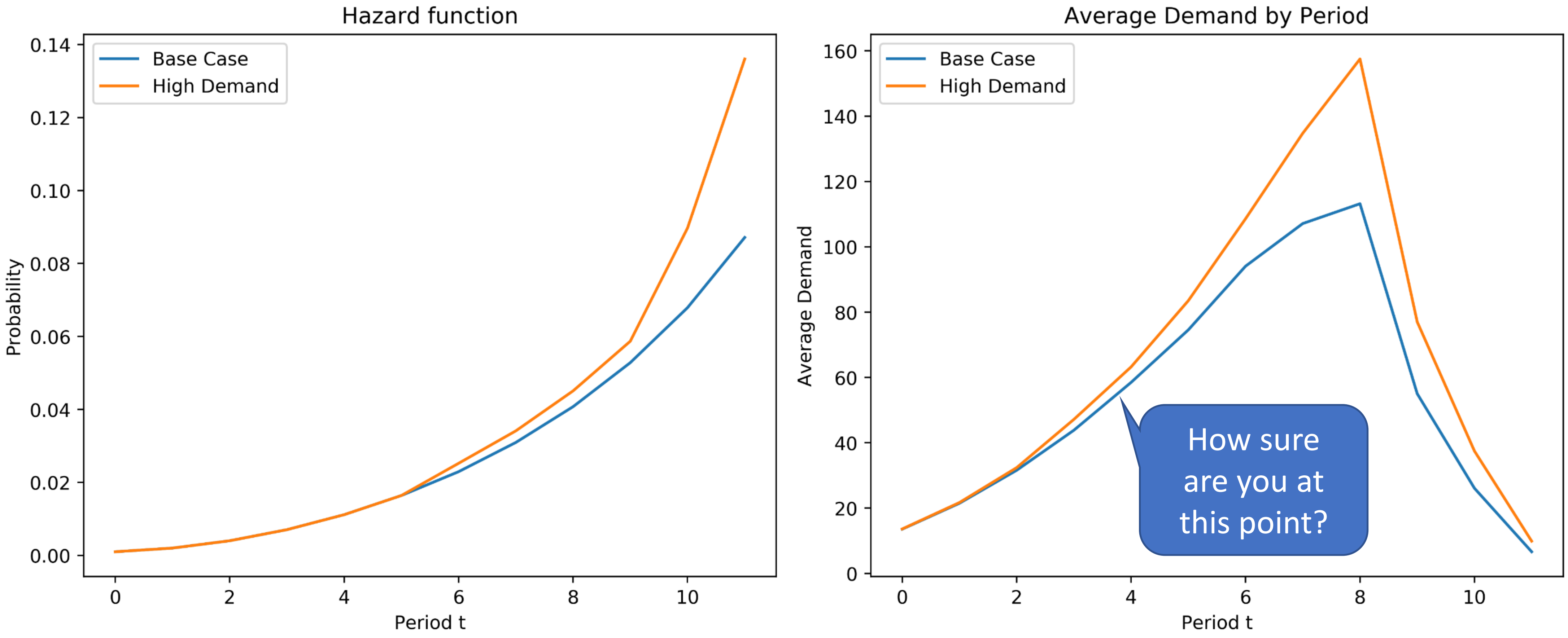

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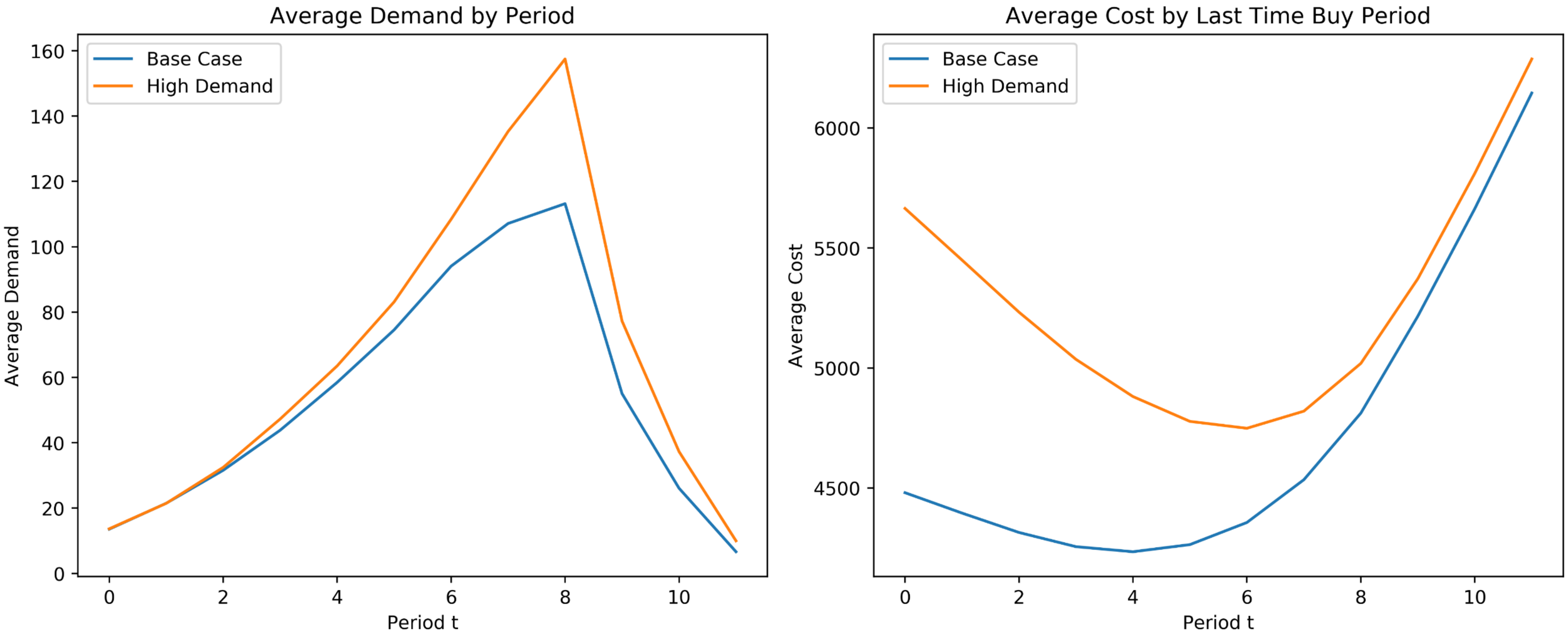

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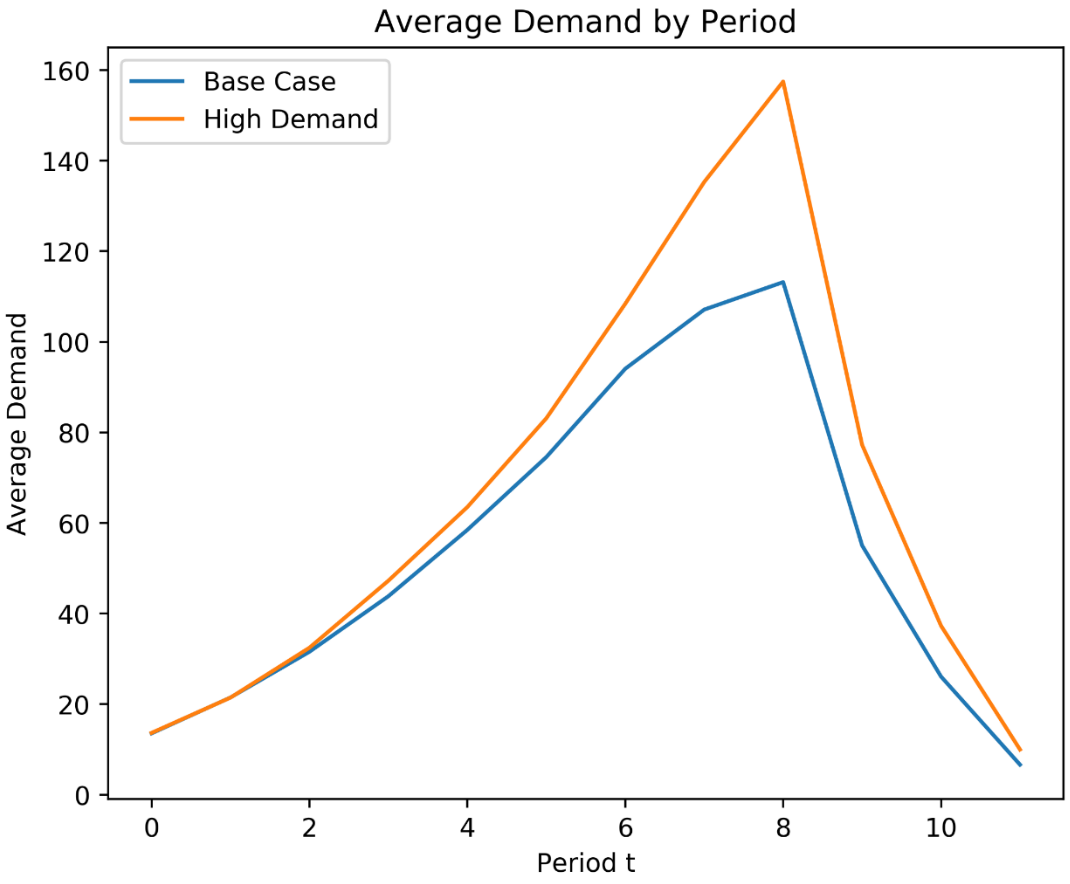


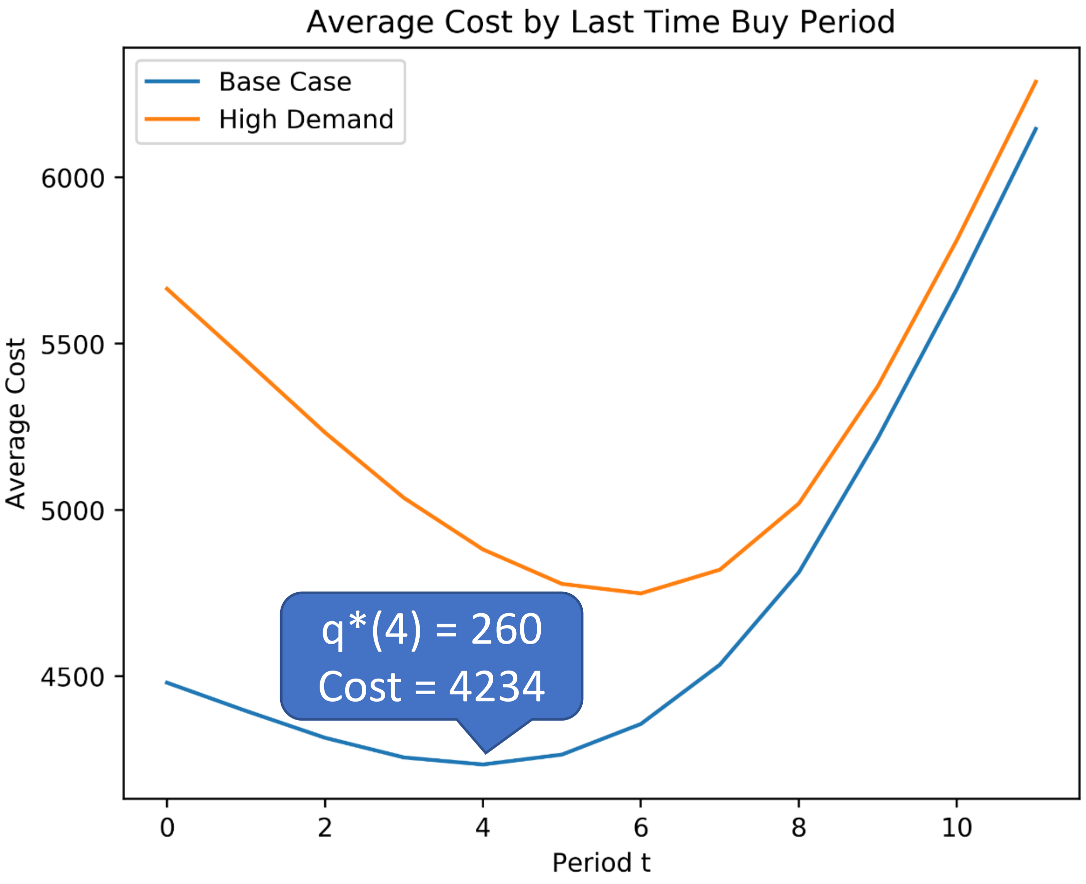

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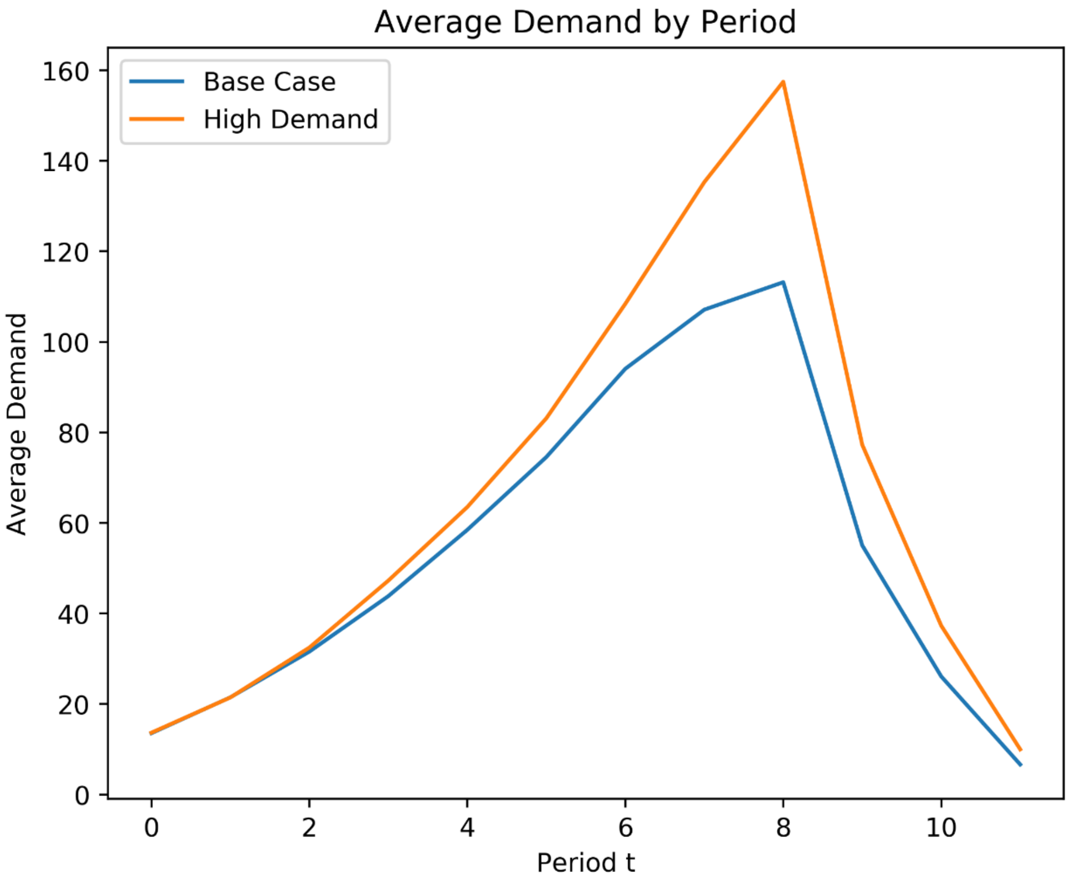


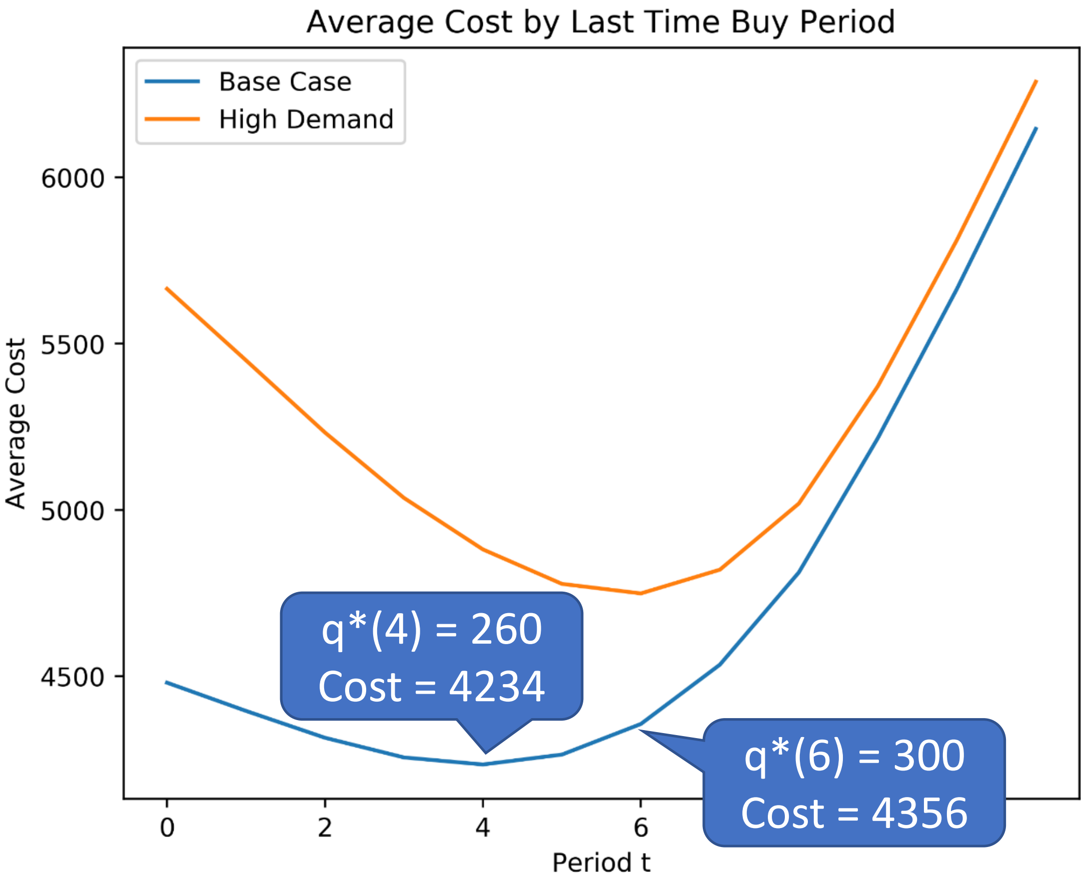

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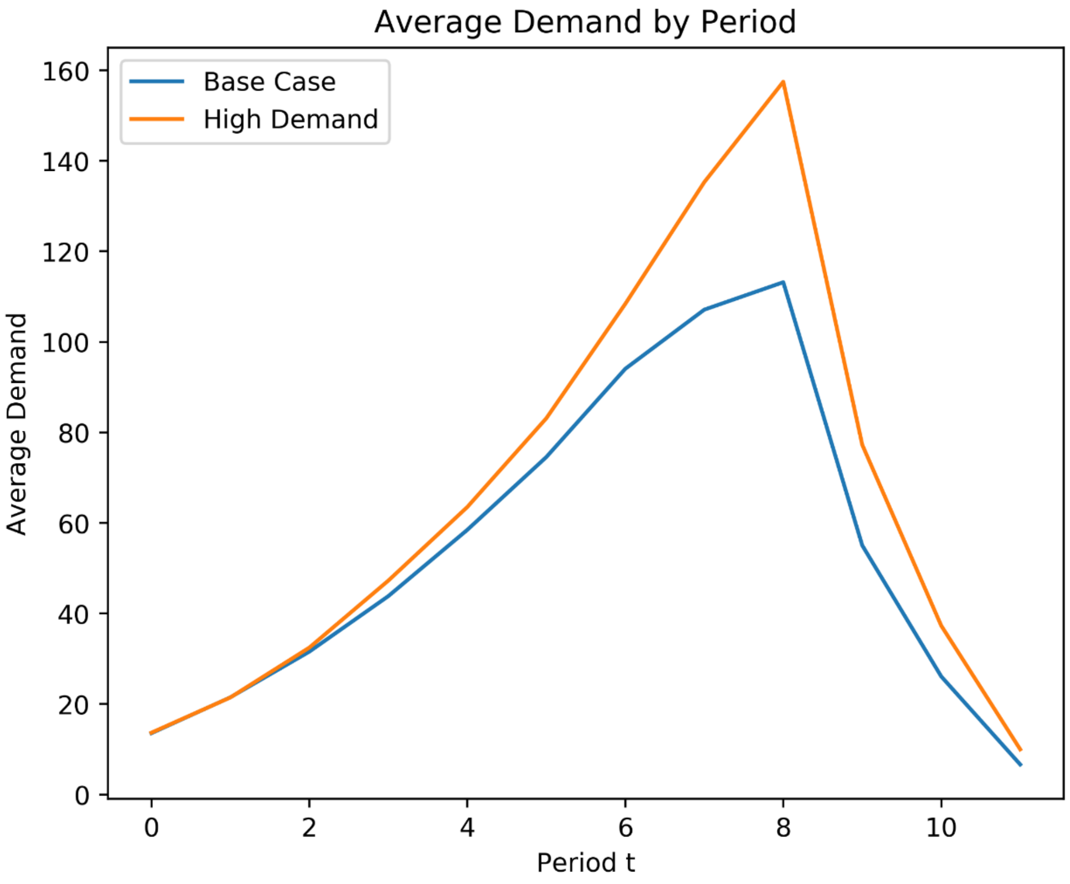


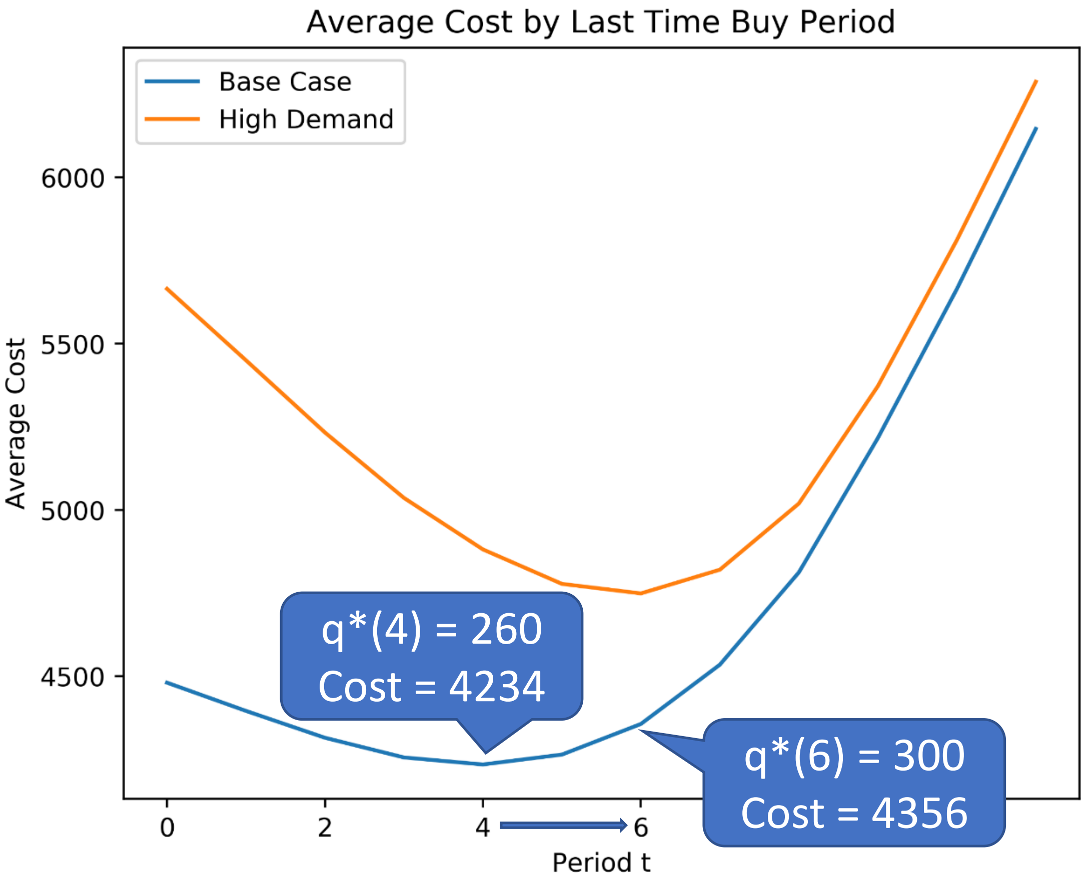


Cost of two period delay: 2.8%

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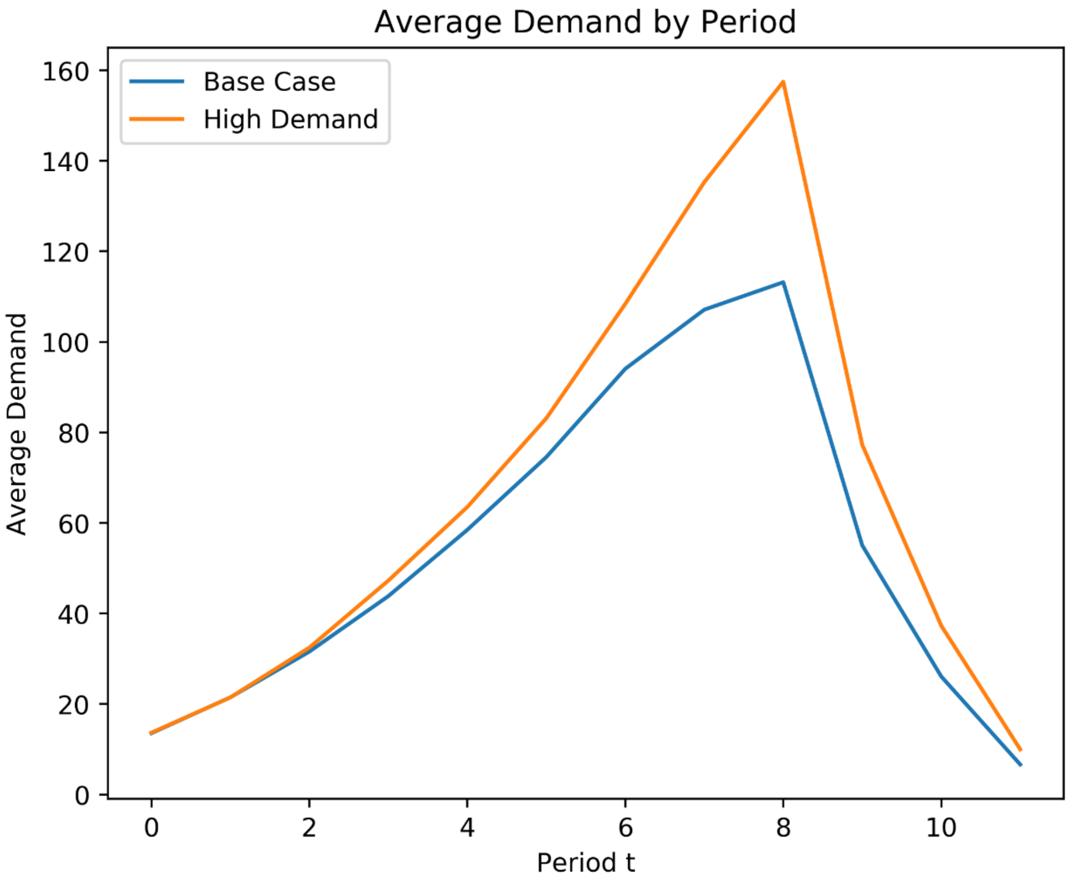
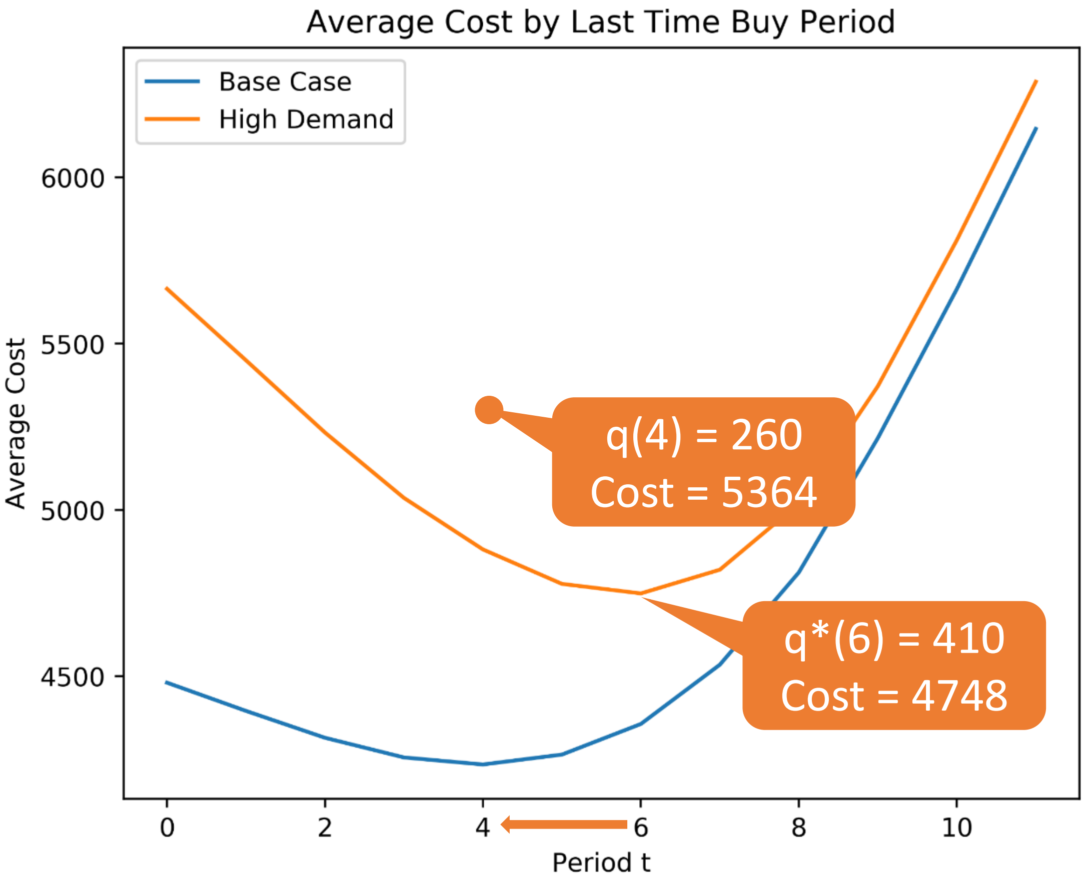
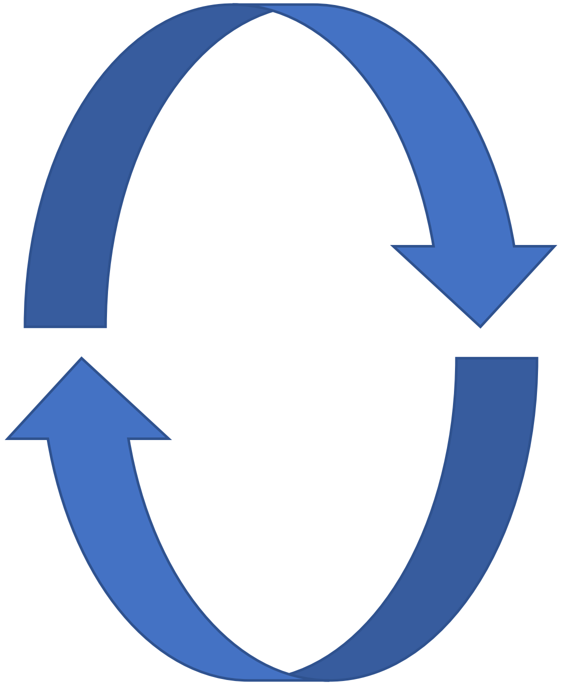



Cost of two period early: 13%

# Forecasting Warranty Claims



Short Product Life Cycles

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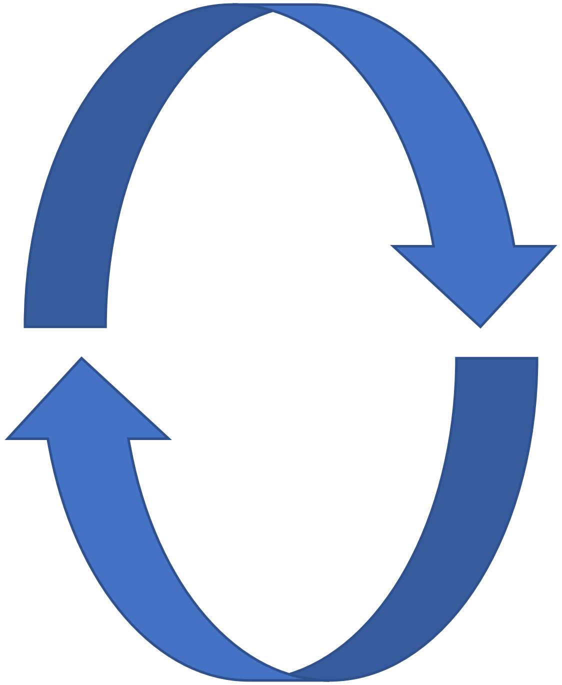

Short Product Life Cycles


Warranty Expiration

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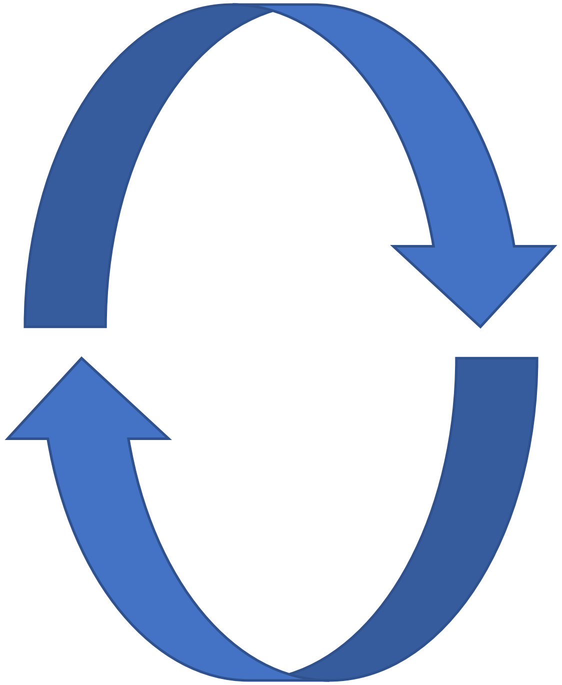

Short Product Life Cycles


Warranty Expiration

Internet of Things

# Thank you

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