Managing Warranty Inventory for Multi-Generational High-Tech Devices

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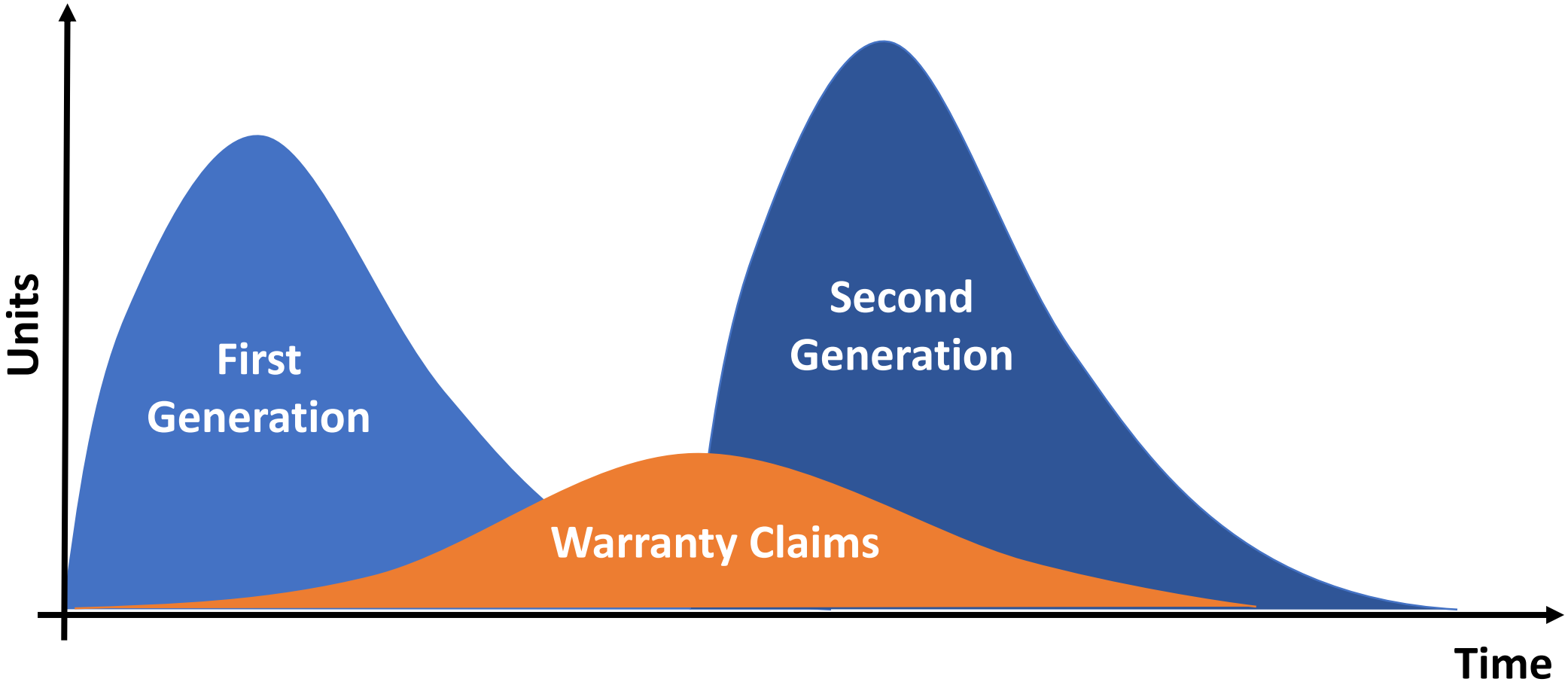
Motivation

Motivation


GARMIN
100
Layup
293
150
Layup
243
Dogleg
194


APPROACH S60
#11 Par 4
397
386
374

When Should We Stop Production?



The Big Questions

Timing

When should we stop production?

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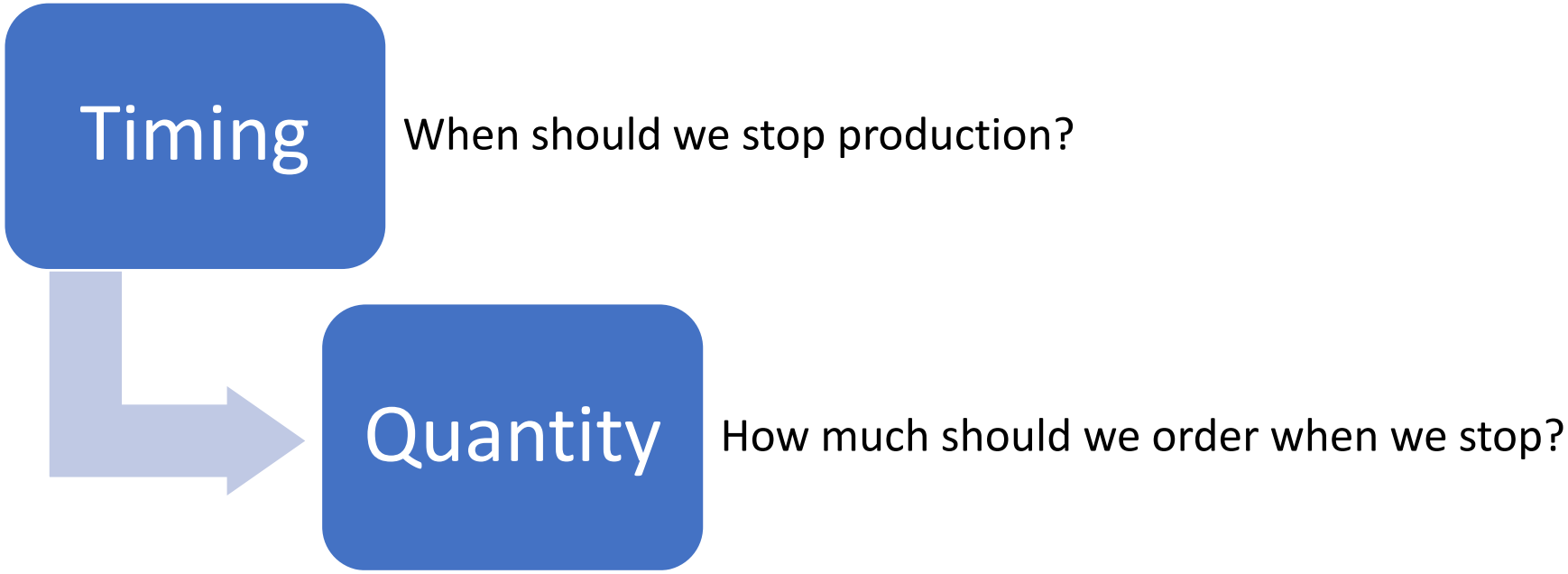

Outline

Literature Review

- Commonly known as:
  - Last Time Buy (LTB)

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- Motivated by spare parts setting
  - Supplier has discontinued an essential component and manufacturer must make LTB

Literature Review

Table 1: Supply Options Considered in Addition to the Last Time Buy

Paper	Repair	Harvest Parts from Returns	Additional Production	Product Trade-Ins
Moore (1971)				
Ritchie and Wilcox (1977)				
Fortuin (1980)				
Fortuin (1981)				
Teunter and Haneveld (1998)				
Teunter and Fortuin (1999)		✓		
Teunter and Haneveld (2002)			✓	
Cattani and Souza (2003)				
Kleber and Inderfurth (2007)		✓		
Inderfurth and Mukherjee (2008)		✓	✓	
Bradley et al. (2009)				
van Kooten and Tan (2009)	✓			
Leifker et al. (2012)			✓	
Pourakbar and Dekker (2012)				
Pourakbar et al. (2012)	✓			
Inderfurth et al. (2013)		✓	✓	
van der Heijden and Iskandar (2013)	✓			
Pourakbar et al. (2014)		✓		✓
Behfard et al. (2015)	✓			
Cole et al. (2015)				✓
Cole et al. (2016)				✓

Assumptions

- We consider only devices that are too costly to repair
- Zero lead time
- Until the final period, warranty claims are satisfied as they arrive
- Warranty claims are
  - Independent period to period
  - From a family of infinitely-divisible distributions (e.g. Normal)
  - Non-negative in each period
- Leftover units have no salvage value

Notation

Parameters

- T - number of periods
- c_p - production cost per unit
- c_s - shortage cost per unit
- c_f - fixed operational production cost per period
- c_h - holding cost per unit per period

Decision Variables

- t - time of final order or final period of production
- q - final order quantity

Notation

Demand Distributions

- D_i - random variable representing demand in period i where $i = 1...T$
- f_i^j - pdf of cumulative demand from period i to period j
- F_i^j - cdf of cumulative demand from period i to period j

Expected Cost

$$\min_{t,q} \quad c_f t$$

Operational Costs

Expected Cost

$$\min_{t,q} \quad c_f t + c_p \sum_{i=1}^{t} \mathbb{E}[D_i] + c_p q$$

Operational Costs Production Costs

Expected Cost

$$\min_{t,q} \quad c_f t + c_p \sum_{i=1}^{t} \mathbb{E}[D_i] + c_p q + c_h \sum_{i=t+1}^{T} \mathbb{E}\left[\left(q - \sum_{j=t+1}^{i} D_j\right)^+\right]$$

Operational Costs Production Costs Holding Costs

Expected Cost

$$\min_{t,q} \quad \underbrace{c_f t}_{\text{Operational Costs}} + \underbrace{c_p \sum_{i=1}^{t} \mathbb{E}[D_i] + c_p q}_{\text{Production Costs}} + \underbrace{c_h \sum_{i=t+1}^{T} \mathbb{E}\left[\left(q - \sum_{j=t+1}^{i} D_j\right)^+\right]}_{\text{Holding Costs}} + \underbrace{c_s \, \mathbb{E}\left[\left(\sum_{i=t+1}^{T} D_i - q\right)^+\right]}_{\text{Shortage Costs}}$$

First and Second Order Conditions

Consider finding the optimal q associated with a fixed t

$$c_p + c_h \sum_{i=t+1}^{T} F_{t+1}^{i}(q) + c_s(F_{t+1}^{T}(q) - 1) = 0$$

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Let $q^*(t)$ represent the implicit solution to the FONC

Modified Objective

$$\min_{t} \quad c_f t + c_p \sum_{i=1}^{t} \mathbb{E}[D_i] - c_h \sum_{i=t+1}^{T} \int_0^{q^*(t)} x f_{t+1}^{i}(x) dx + c_s \int_{q^*(t)}^{\infty} x f_{t+1}^{T}(x) dx$$

Solution Properties

The Expected Cost is convex in q for a given t

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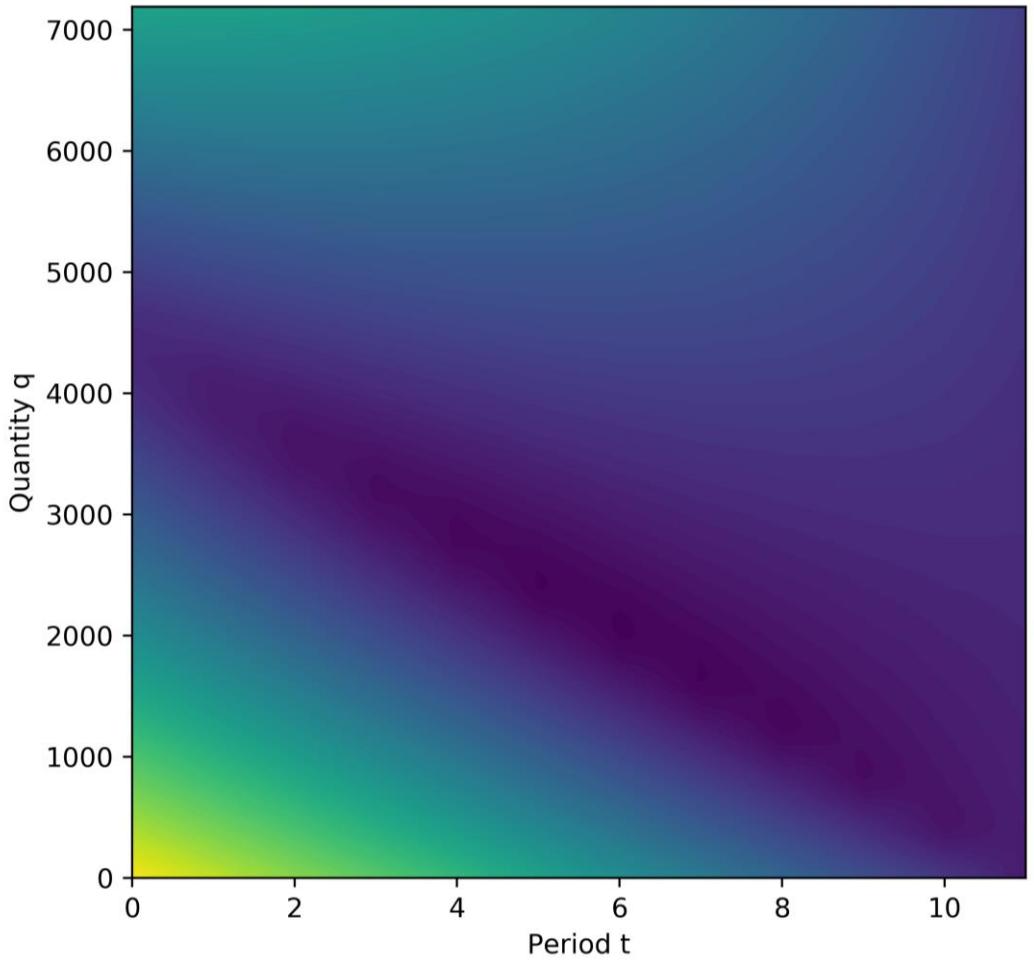
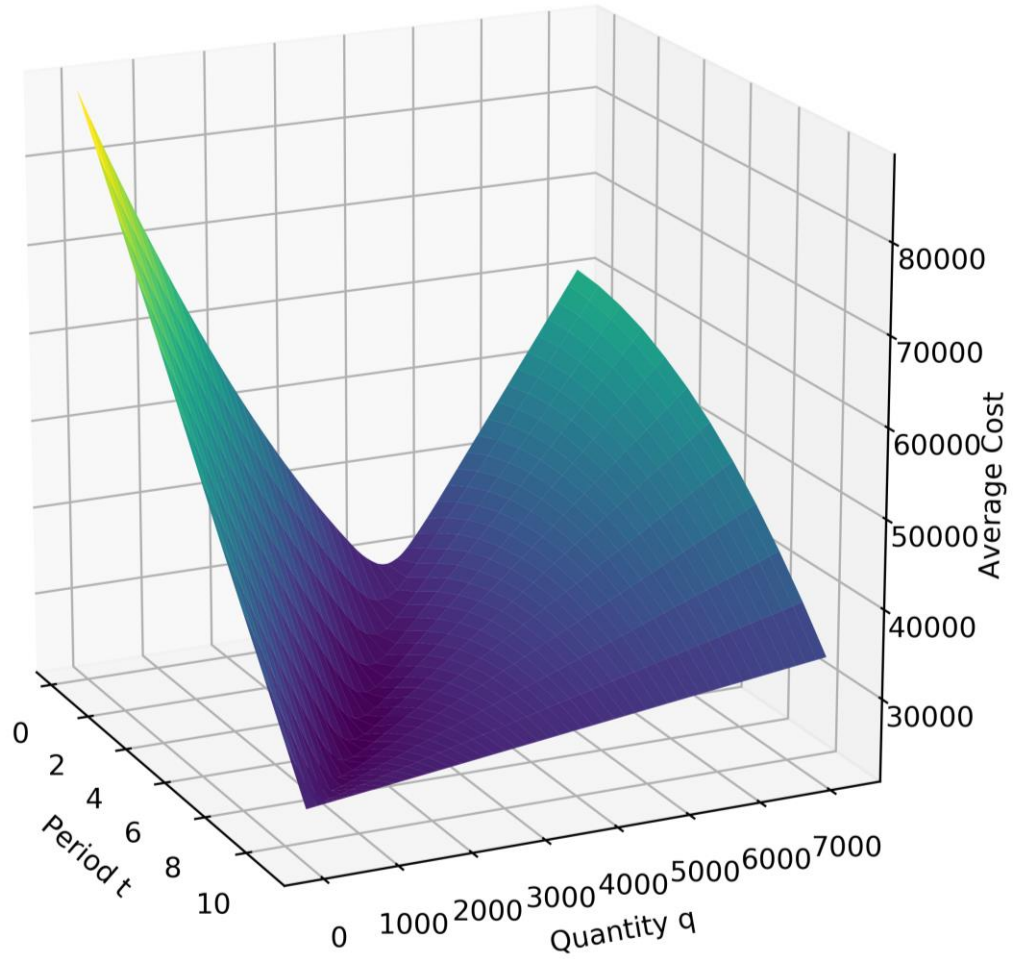
$q^*(t)$ is non-increasing in t

Simulation Results

Average Cost for a Variety of Stopping Periods and Order Quantities





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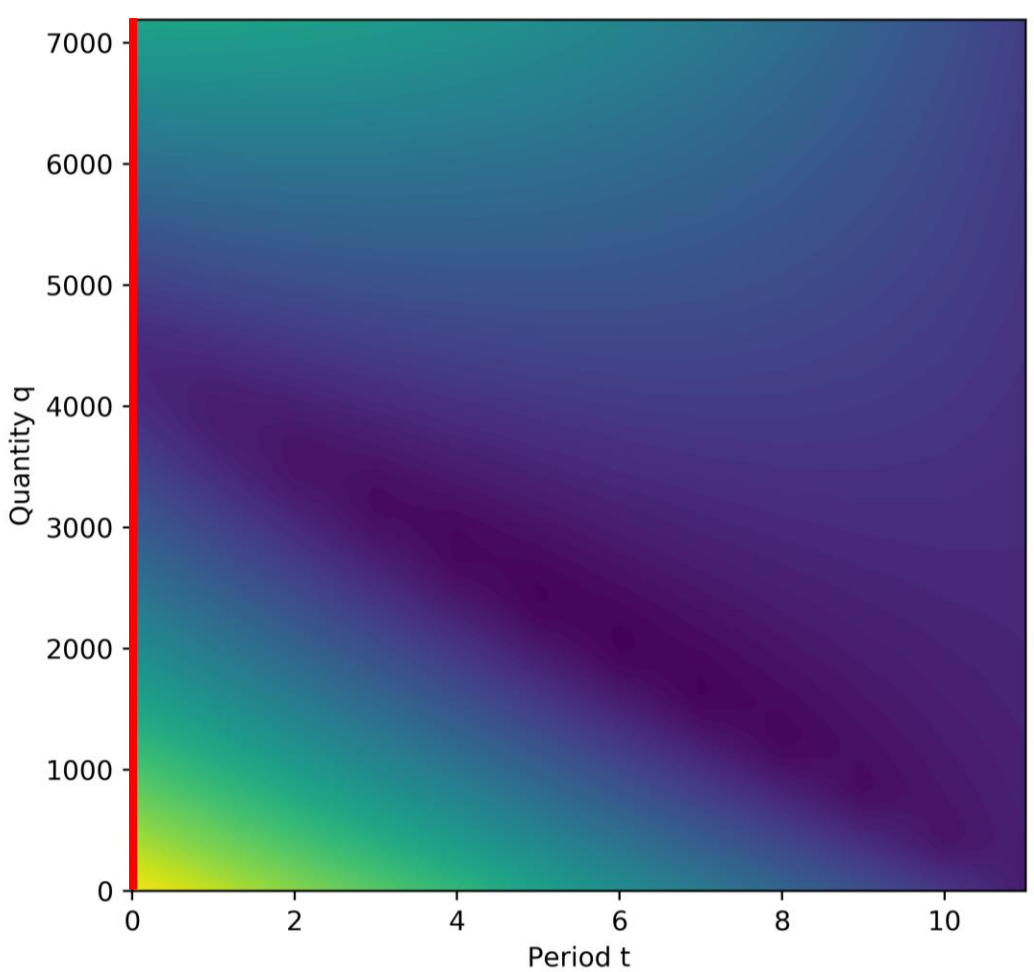

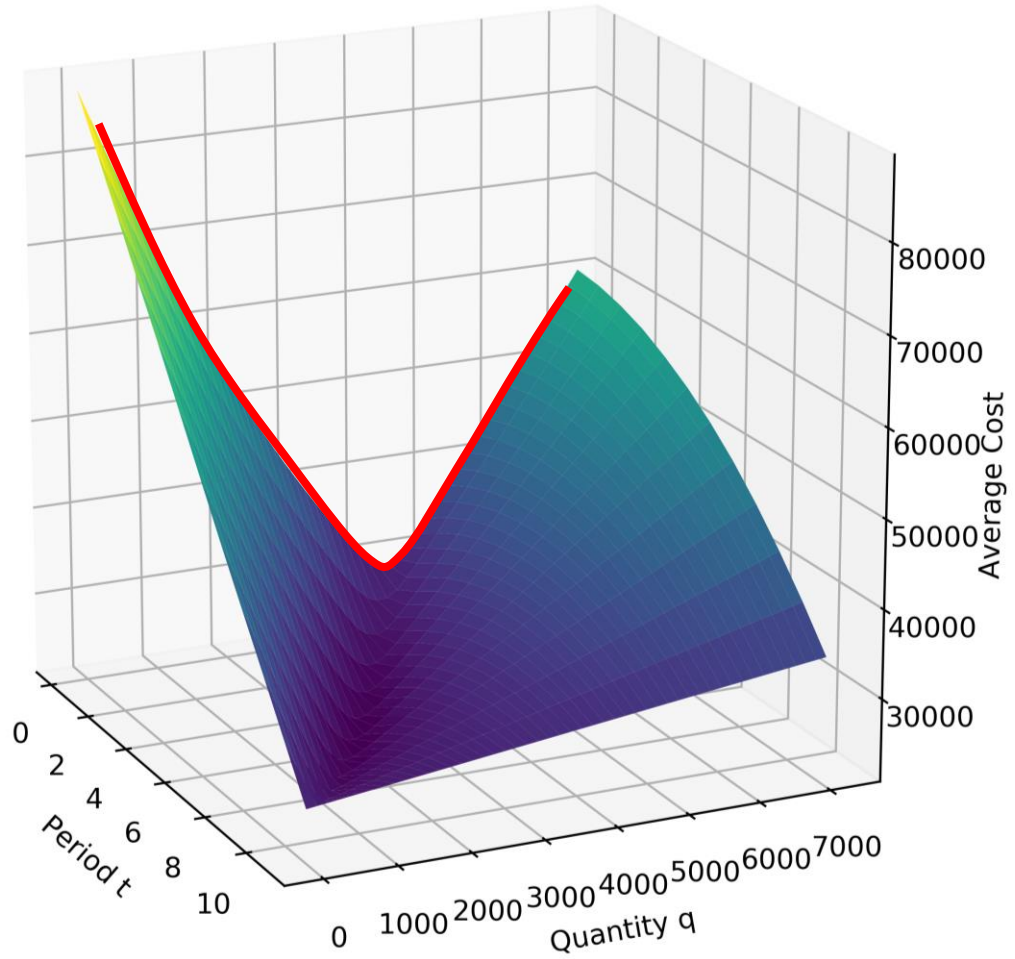

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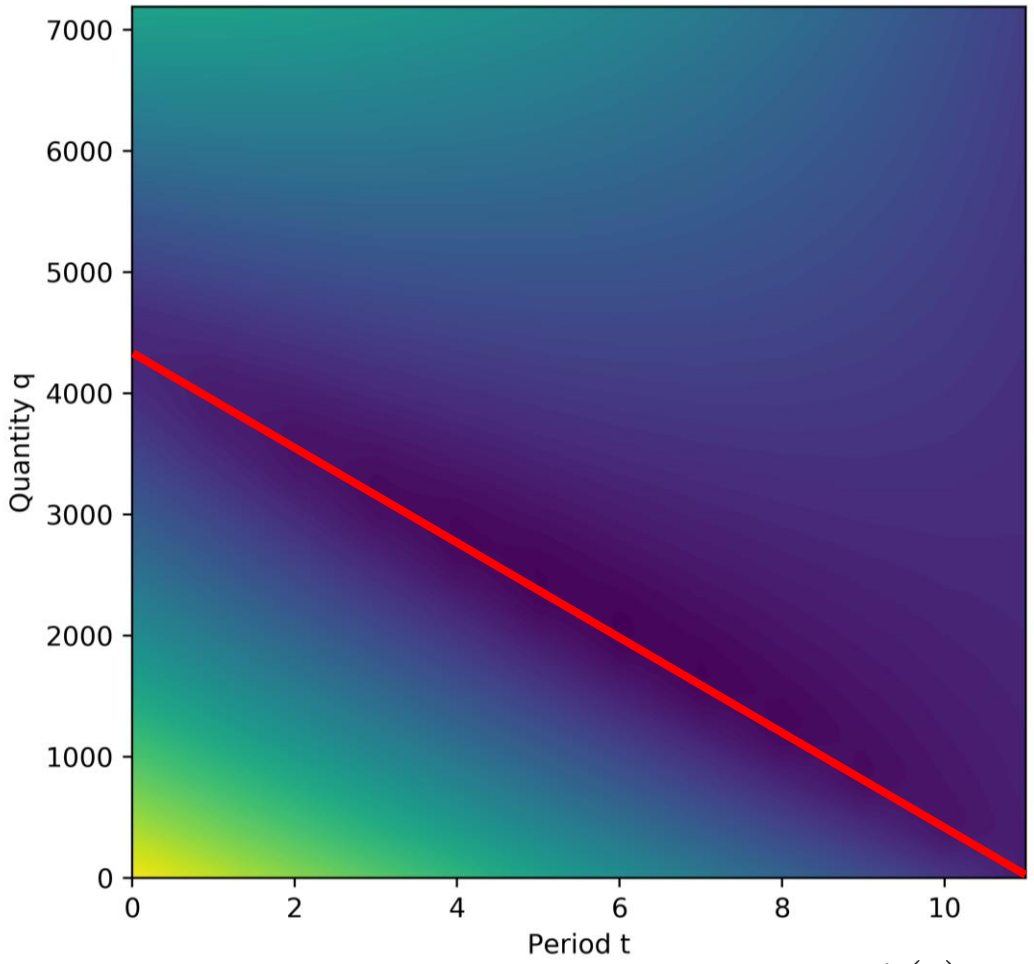

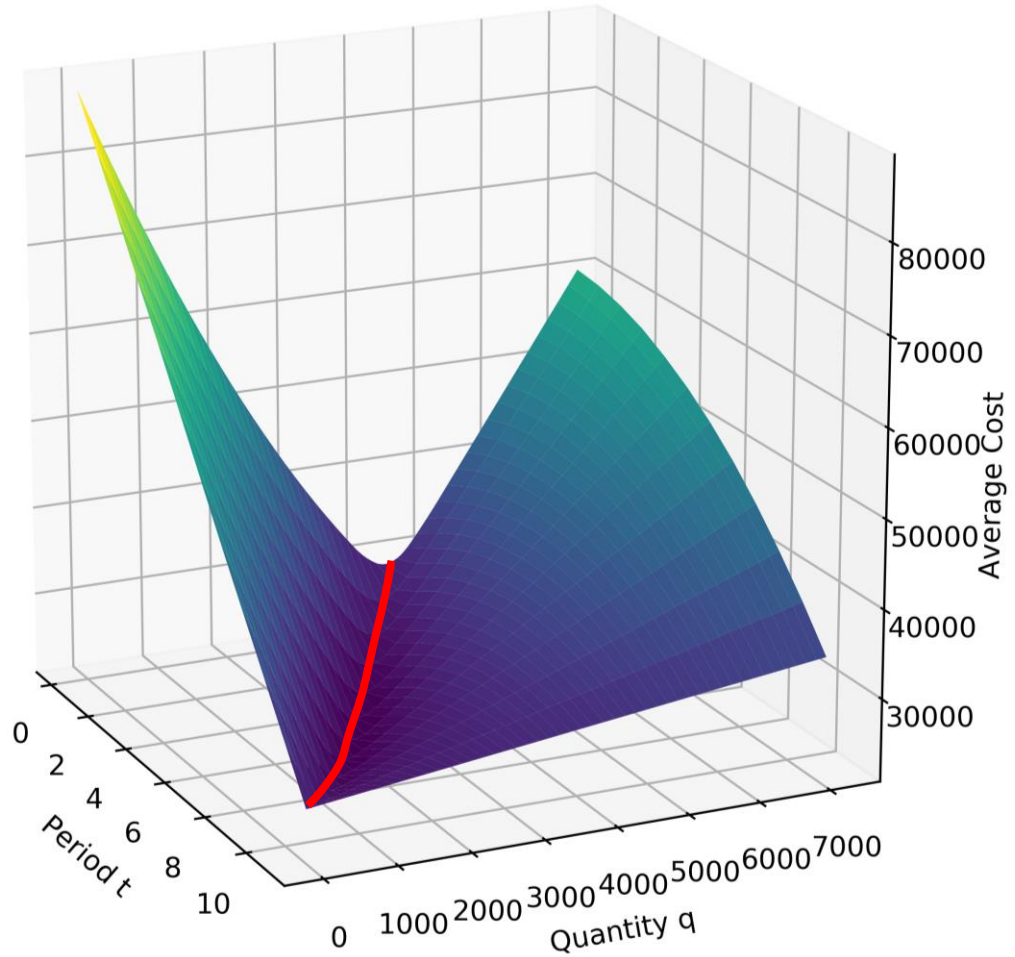

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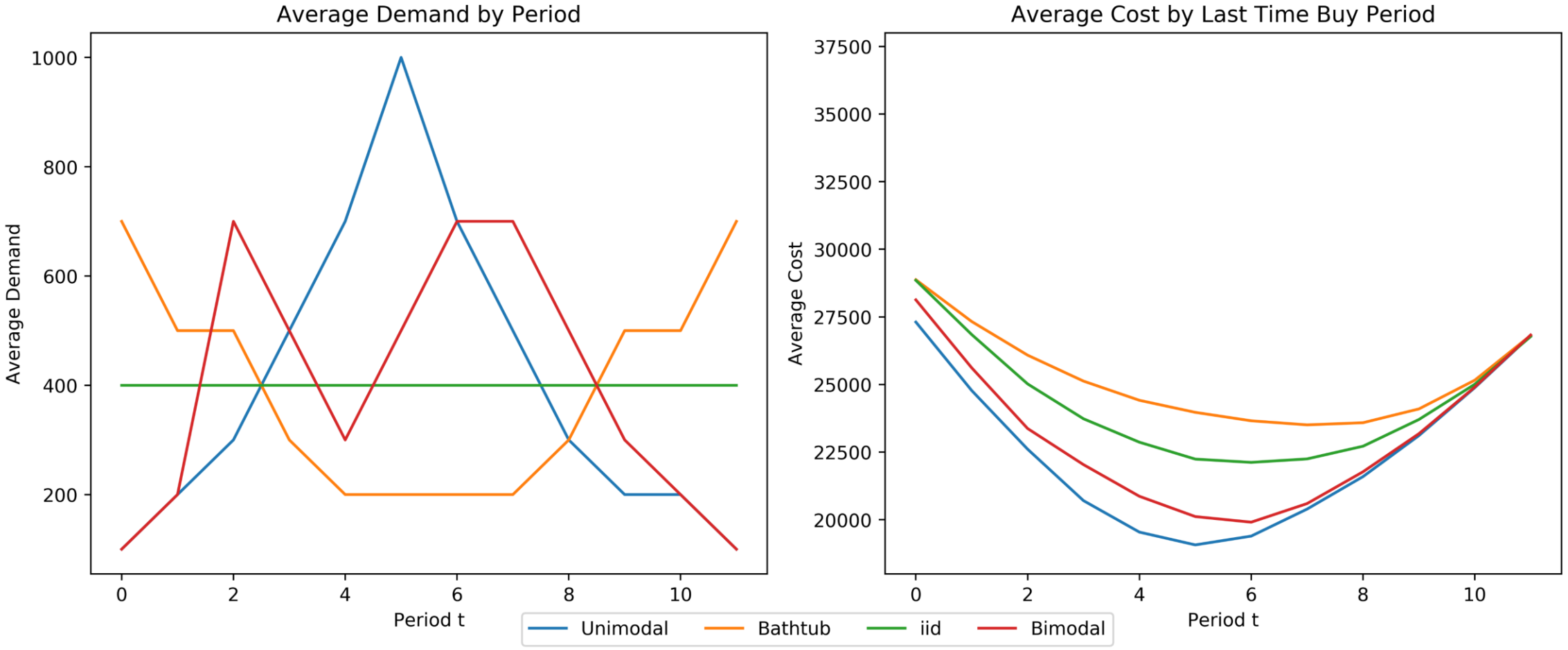

Moral Hazard

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How big of a danger is this moral hazard? It depends on:

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I just sent in my broken golf watch for repair, and the company sent me a brand new golf watch 2.0 instead!

How big of a danger is this moral hazard? It depends on:

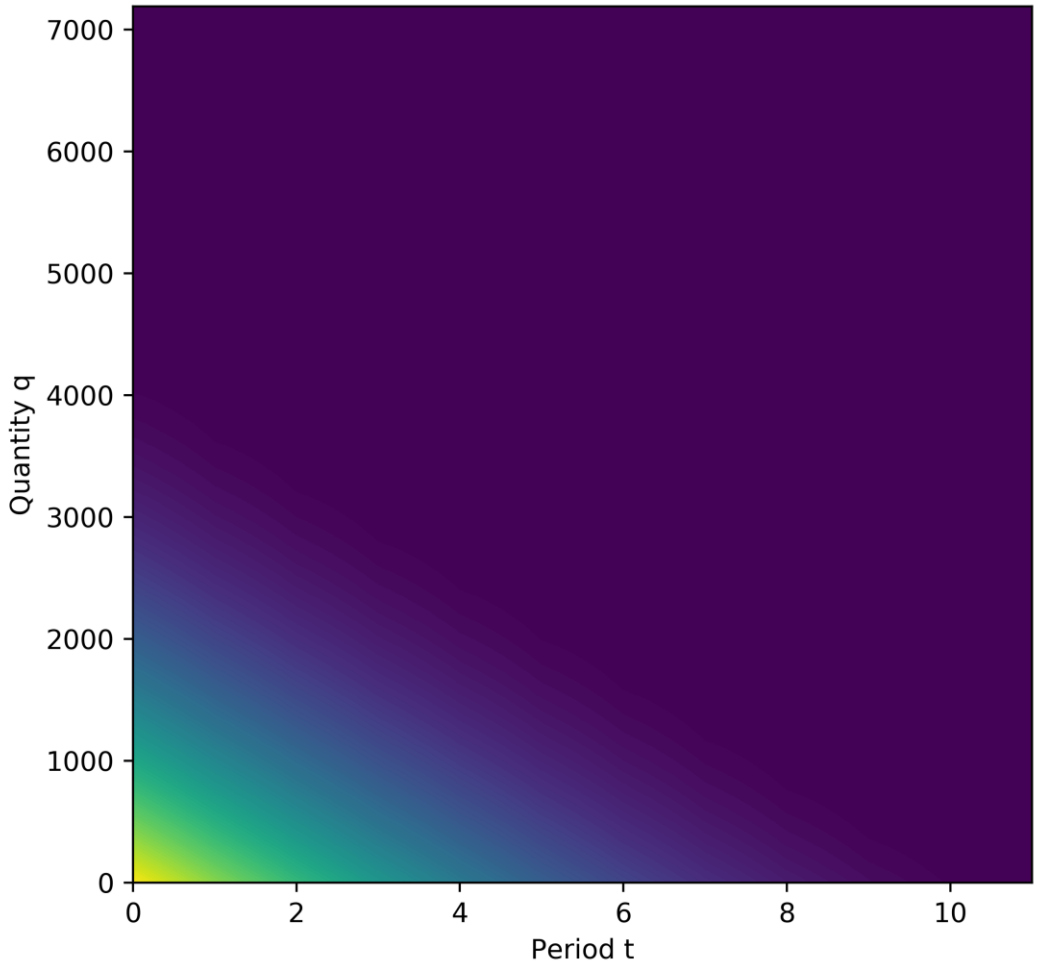
1. The number of new devices given out
2. The time relative to the new product introduction

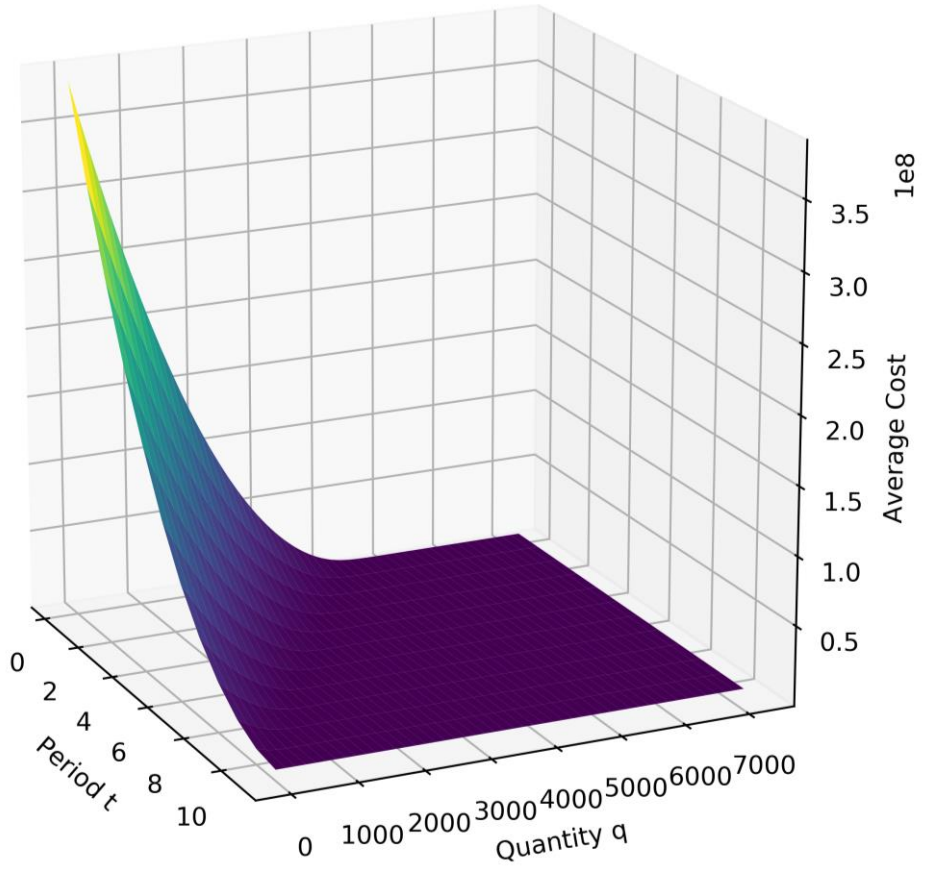
Quadratic Shortages

$$\min_{t,q} \quad c_f t + c_p \sum_{i=1}^{t} \mathbb{E}[D_i] + c_p q + c_h \sum_{i=t+1}^{T} \mathbb{E}\left[\left(q - \sum_{j=t+1}^{i} D_j\right)^+\right] + c_s \, \mathbb{E}\left[\left(\left(\sum_{i=t+1}^{T} D_i - q\right)^+\right)^2\right]$$

Quadratic Shortages

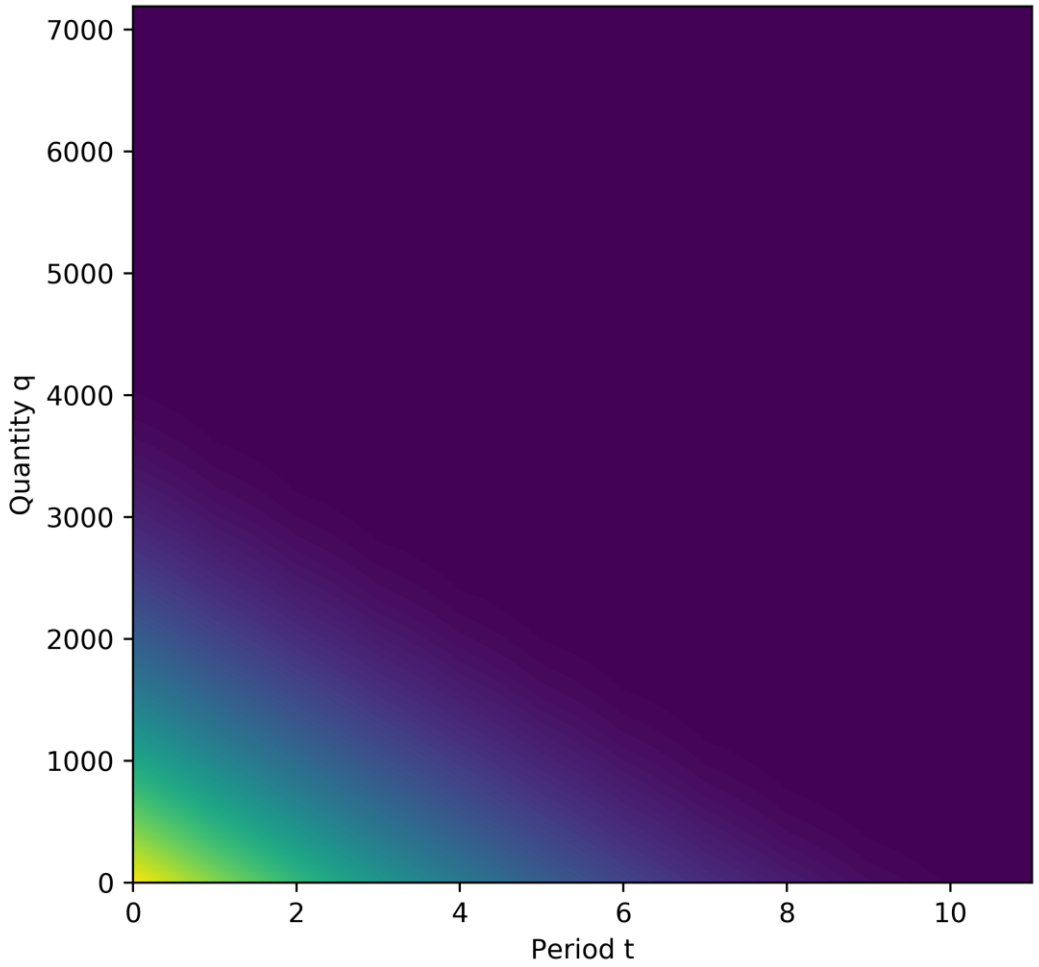
Average Cost for a Variety of Stopping Periods and Order Quantities

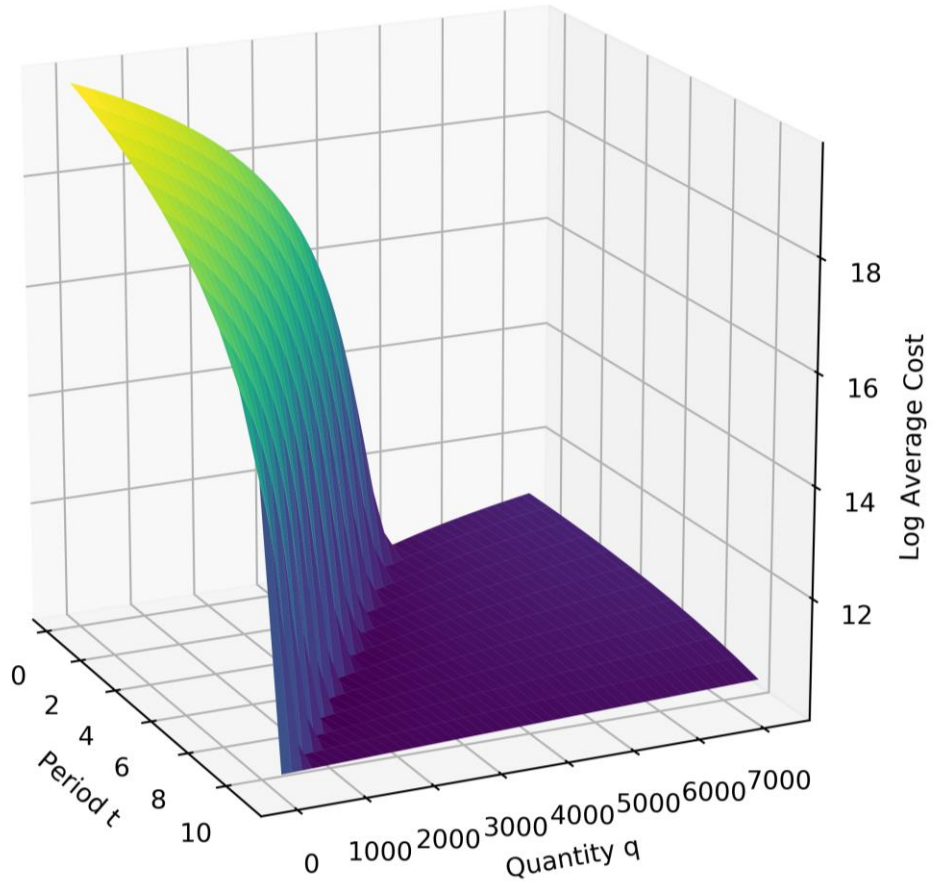




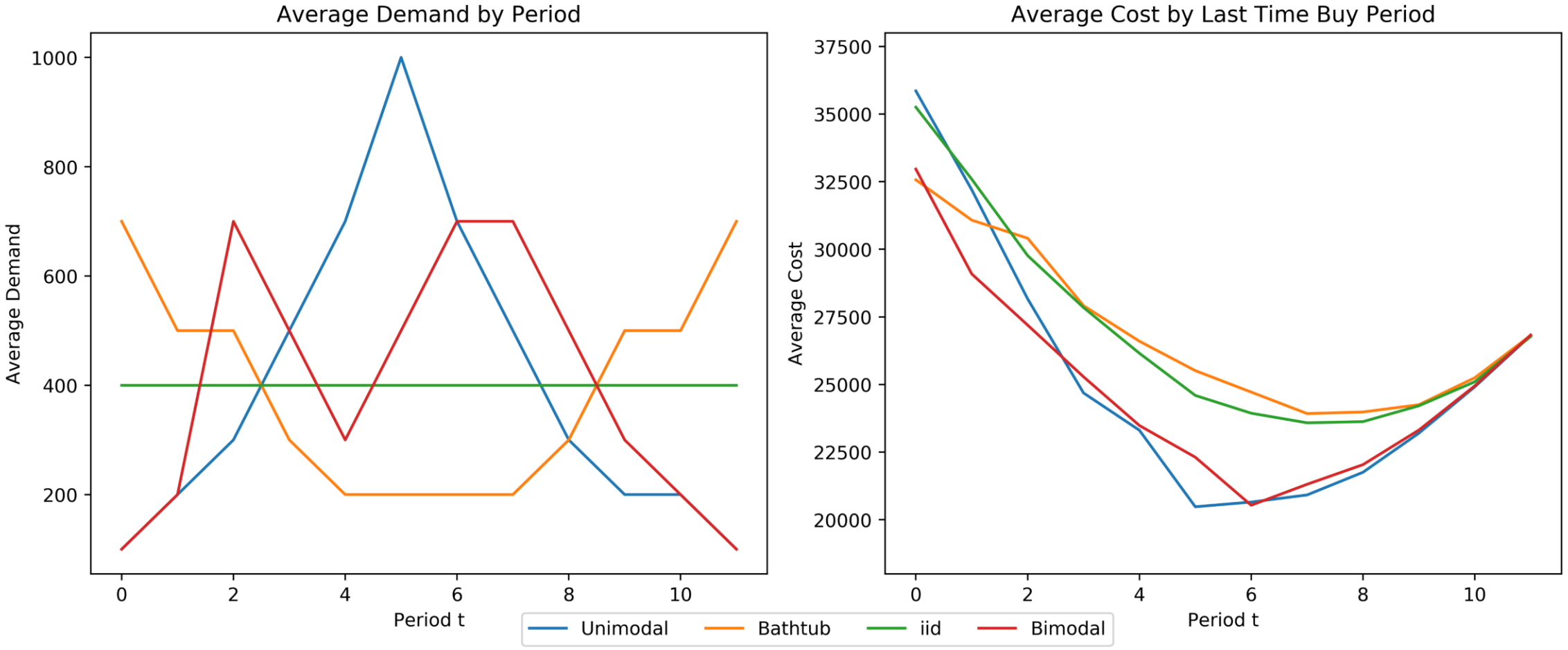
Quadratic Shortages

Log of Average Cost for a Variety of Stopping Periods and Order Quantities





Simulation Results



Forecasting Warranty Claims

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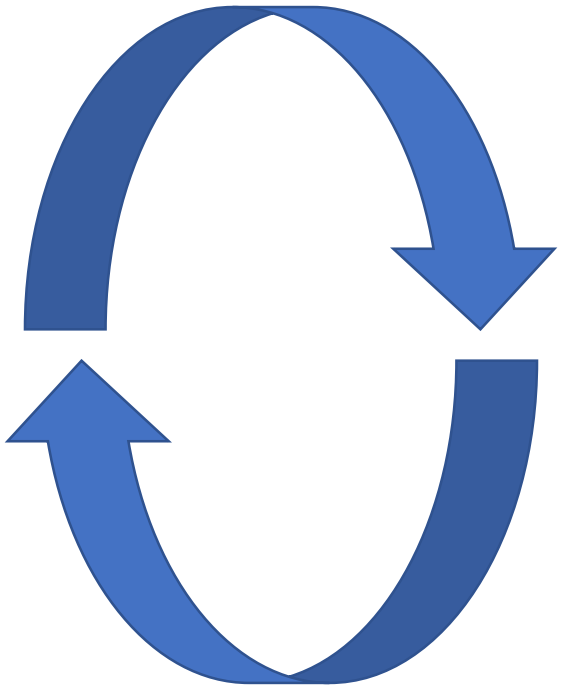
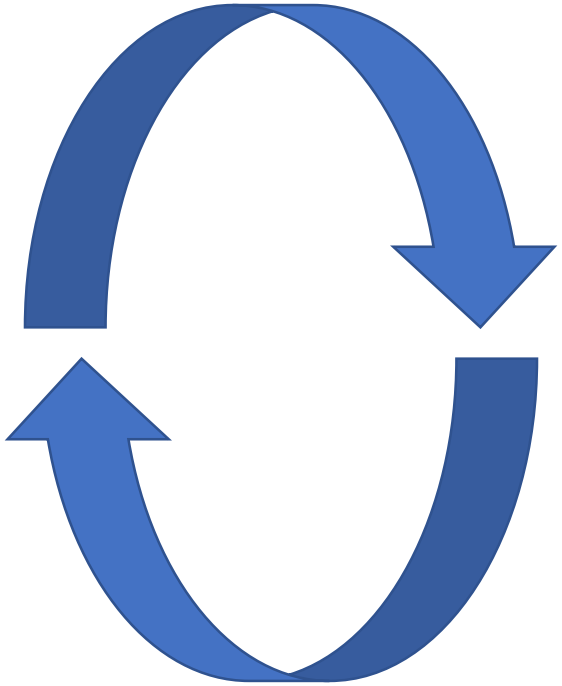


Short Product Life Cycles

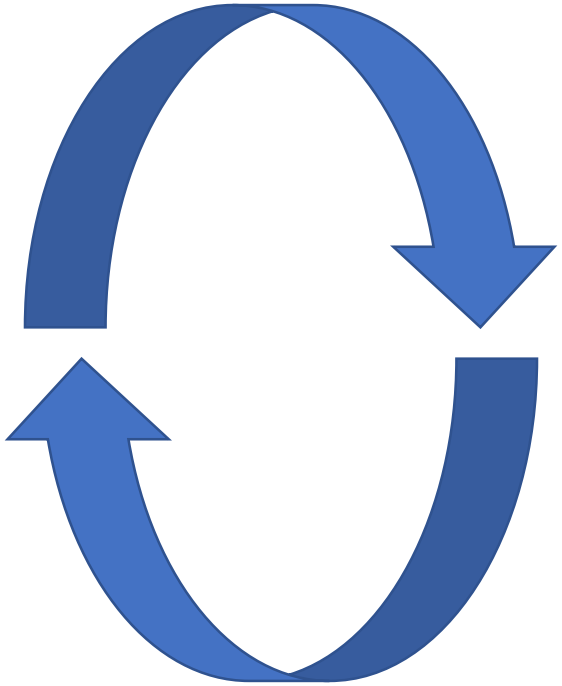
Forecasting Warranty Claims



Short Product Life Cycles

Warranty Expiration

Forecasting Warranty Claims



Short Product Life Cycles

Warranty Expiration

Internet of Things

Thank you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