Managing Warranty Inventory for Multi-Generational High-Tech Devices

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INDUSTRIAL ENGINEERING & OPERATIONS RESEARCH

Motivation

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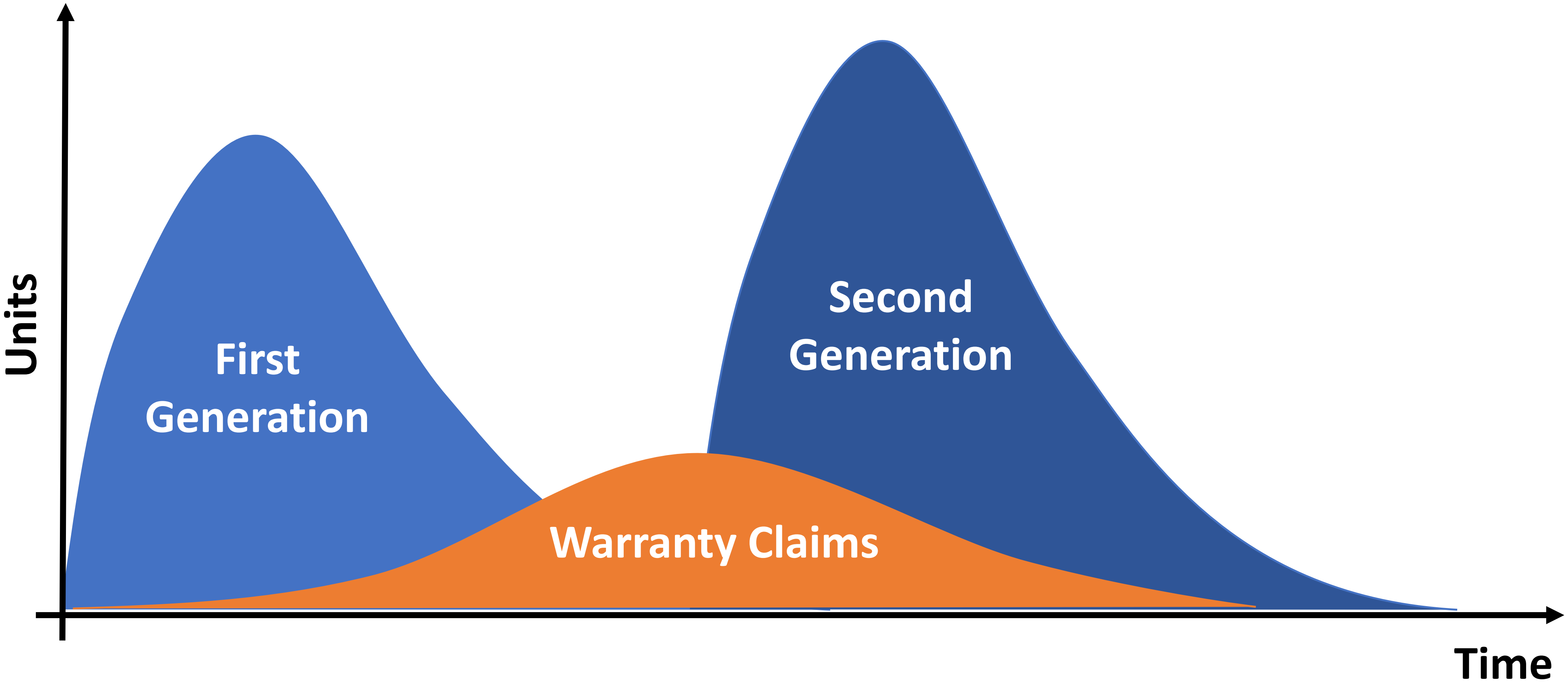
GARMIN
100 Layup 293
150 Layup 243
Dogleg 194


APPROACH S60
#11 Par 4
397
386
374

When Should We Stop Production?



The Big Questions

Timing

When should we stop production?

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Timing — When should we stop production?

Quantity — How much should we order when we stop?

Outline

Literature Review → Model → Simulations → Future Work

Literature Review

- Commonly known as:
  - Last Time Buy (LTB)

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- Motivated by spare parts setting
  - Supplier has discontinued an essential component and manufacturer must make LTB

Literature Review

Table 1: Supply Options Considered in Addition to the Last Time Buy

Paper	Repair	Harvest Parts from Returns	Additional Production	Product Trade-Ins
Moore (1971)				
Ritchie and Wilcox (1977)				
Fortuin (1980)				
Fortuin (1981)				
Teunter and Haneveld (1998)				
Teunter and Fortuin (1999)		✓		
Teunter and Haneveld (2002)			✓	
Cattani and Souza (2003)				
Kleber and Inderfurth (2007)		✓		
Inderfurth and Mukherjee (2008)		✓	✓	
Bradley et al. (2009)				
van Kooten and Tan (2009)	✓			
Leifker et al. (2012)			✓	
Pourakbar and Dekker (2012)				
Pourakbar et al. (2012)	✓			
Inderfurth et al. (2013)		✓	✓	
van der Heijden and Iskandar (2013)	✓			
Pourakbar et al. (2014)		✓		✓
Behfard et al. (2015)	✓			
Cole et al. (2015)				✓
Cole et al. (2016)				✓

Assumptions

- We consider only devices that are too costly to repair
- Zero lead time
- Until the final period, warranty claims are satisfied as they arrive
- Warranty claims are
  - Independent period to period
  - From a family of infinitely-divisible distributions (e.g. Normal)
  - Non-negative in each period
- Leftover units have no salvage value

Notation

Parameters

- T - number of periods
- c_p - production cost per unit
- c_s - shortage cost per unit
- c_f - fixed operational production cost per period
- c_h - holding cost per unit per period

Decision Variables

- t - time of final order or final period of production
- q - final order quantity

Notation

Demand Distributions

- D_i - random variable representing demand in period i where $i = 1...T$
- f_i^j - pdf of cumulative demand from period i to period j
- F_i^j - cdf of cumulative demand from period i to period j

Expected Cost

$$\min_{t,q} \quad \underbrace{c_f t}_{\text{Operational Costs}}$$

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$$\min_{t,q} \quad \underbrace{c_f t}_{\text{Operational Costs}} + \underbrace{c_p \sum_{i=1}^{t} \mathbb{E}[D_i] + c_p q}_{\text{Production Costs}}$$

Operational Costs Production Costs

Expected Cost

$$\min_{t,q} \quad c_f t + c_p \sum_{i=1}^{t} \mathbb{E}[D_i] + c_p q + c_h \sum_{i=t+1}^{T} \mathbb{E}\left[\left(q - \sum_{j=t+1}^{i} D_j\right)^+\right]$$

Operational Costs — Production Costs — Holding Costs

Expected Cost

$$\min_{t,q} \quad \underbrace{c_f t}_{\text{Operational Costs}} + \underbrace{c_p \sum_{i=1}^{t} \mathbb{E}[D_i] + c_p q}_{\text{Production Costs}} + \underbrace{c_h \sum_{i=t+1}^{T} \mathbb{E}\left[\left(q - \sum_{j=t+1}^{i} D_j\right)^+\right]}_{\text{Holding Costs}} + \underbrace{c_s \, \mathbb{E}\left[\left(\sum_{i=t+1}^{T} D_i - q\right)^+\right]}_{\text{Shortage Costs}}$$

First and Second Order Conditions

Consider finding the optimal q associated with a fixed t

$$c_p + c_h \sum_{i=t+1}^{T} F_{t+1}^{i}(q) + c_s(F_{t+1}^{T}(q) - 1) = 0$$

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$$c_h \sum_{i=t+1}^{T} f_{t+1}^{i}(q) + c_s f_{t+1}^{T}(q) \geq 0$$

Let $q^*(t)$ represent the implicit solution to the FONC

Modified Objective

$$\min_{t} \quad c_f t + c_p \sum_{i=1}^{t} \mathbb{E}[D_i] - c_h \sum_{i=t+1}^{T} \int_0^{q^*(t)} x f_{t+1}^{i}(x)dx + c_s \int_{q^*(t)}^{\infty} x f_{t+1}^{T}(x)dx$$

Solution Properties

The Expected Cost is convex in q for a given t

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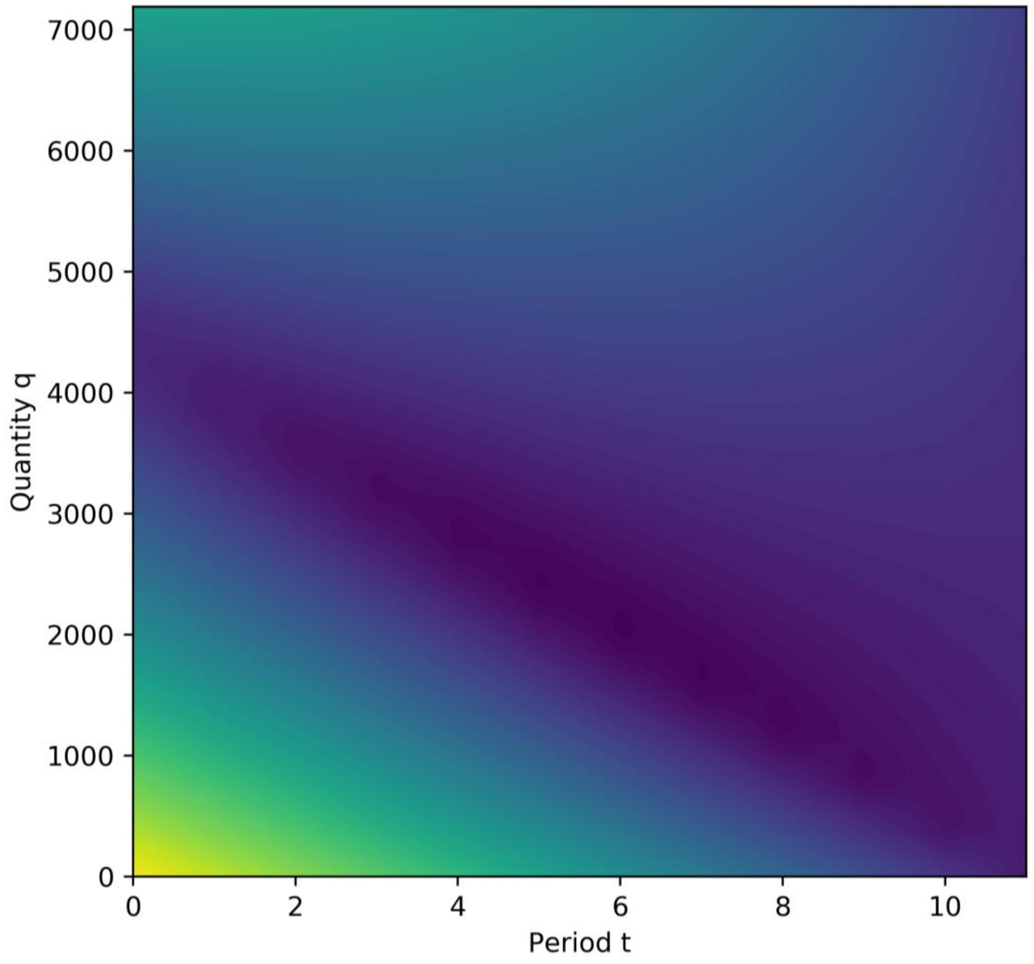
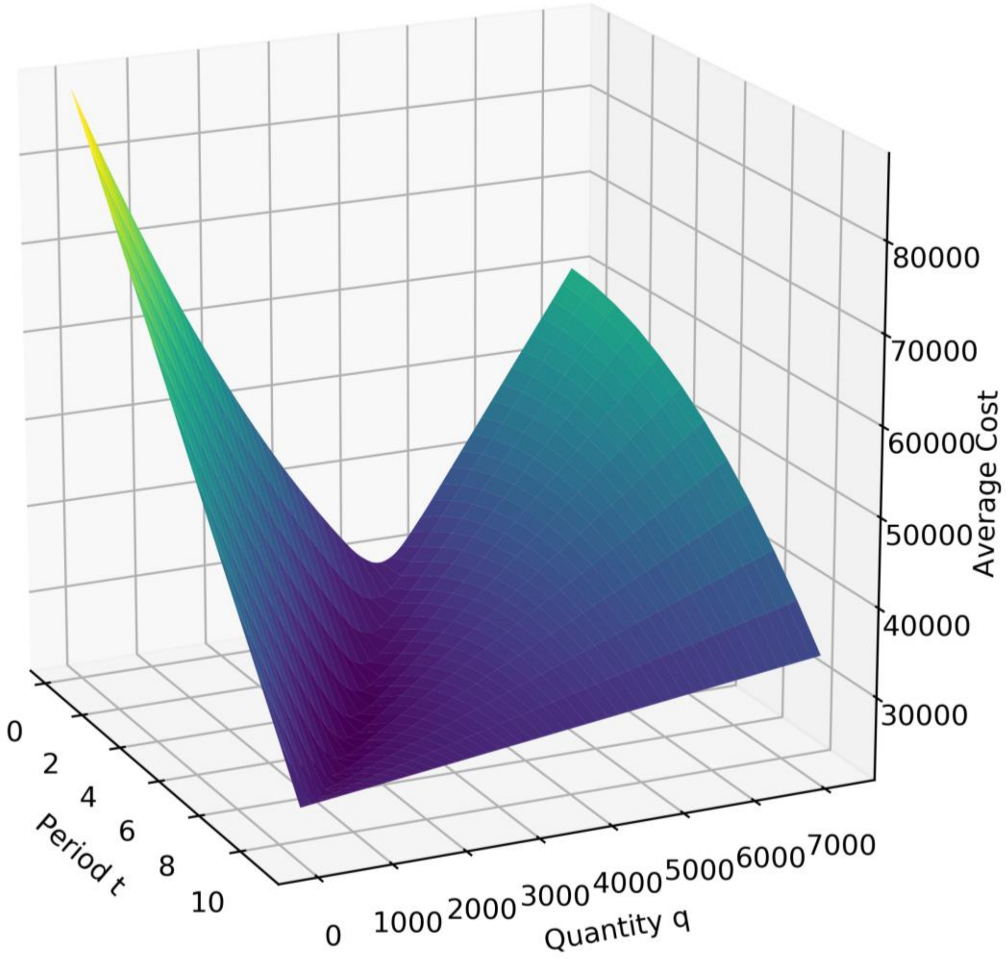
$q^*(t)$ is non-increasing in t

Simulation Results

Average Cost for a Variety of Stopping Periods and Order Quantities





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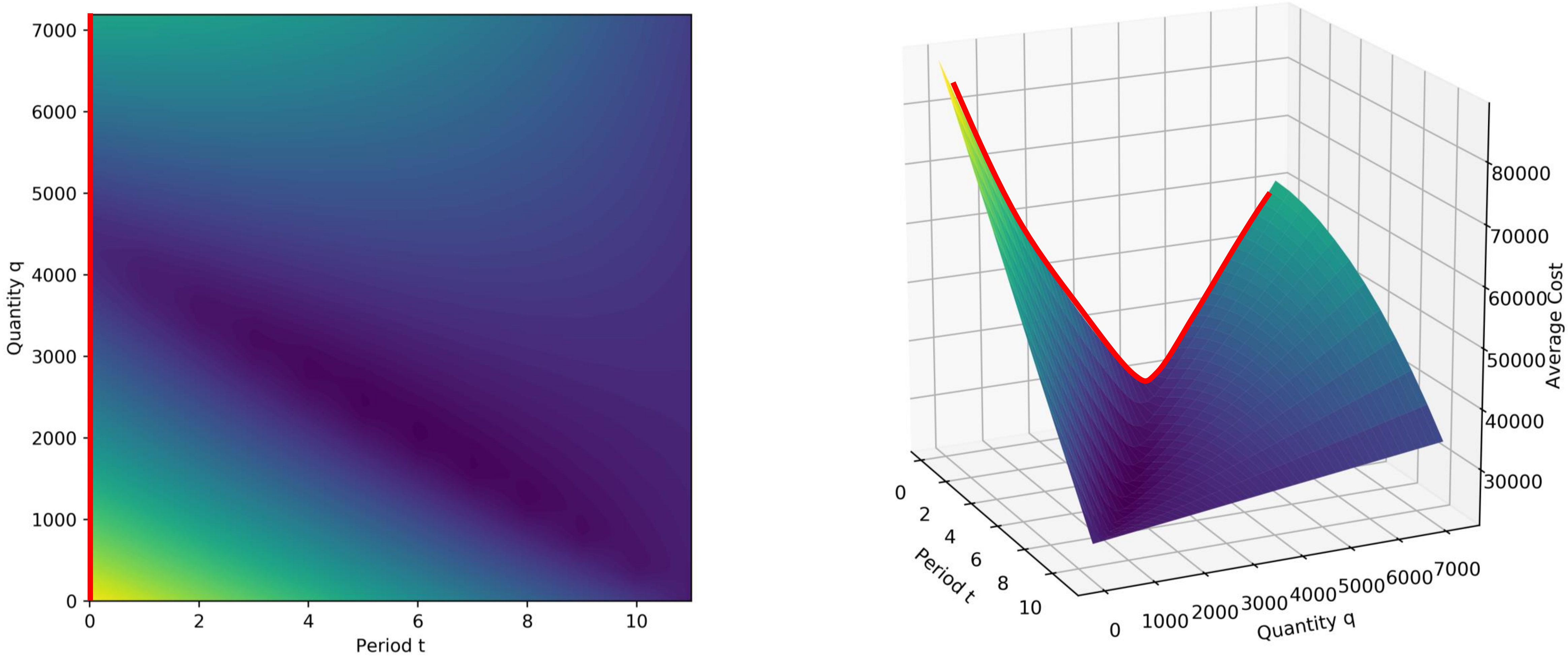

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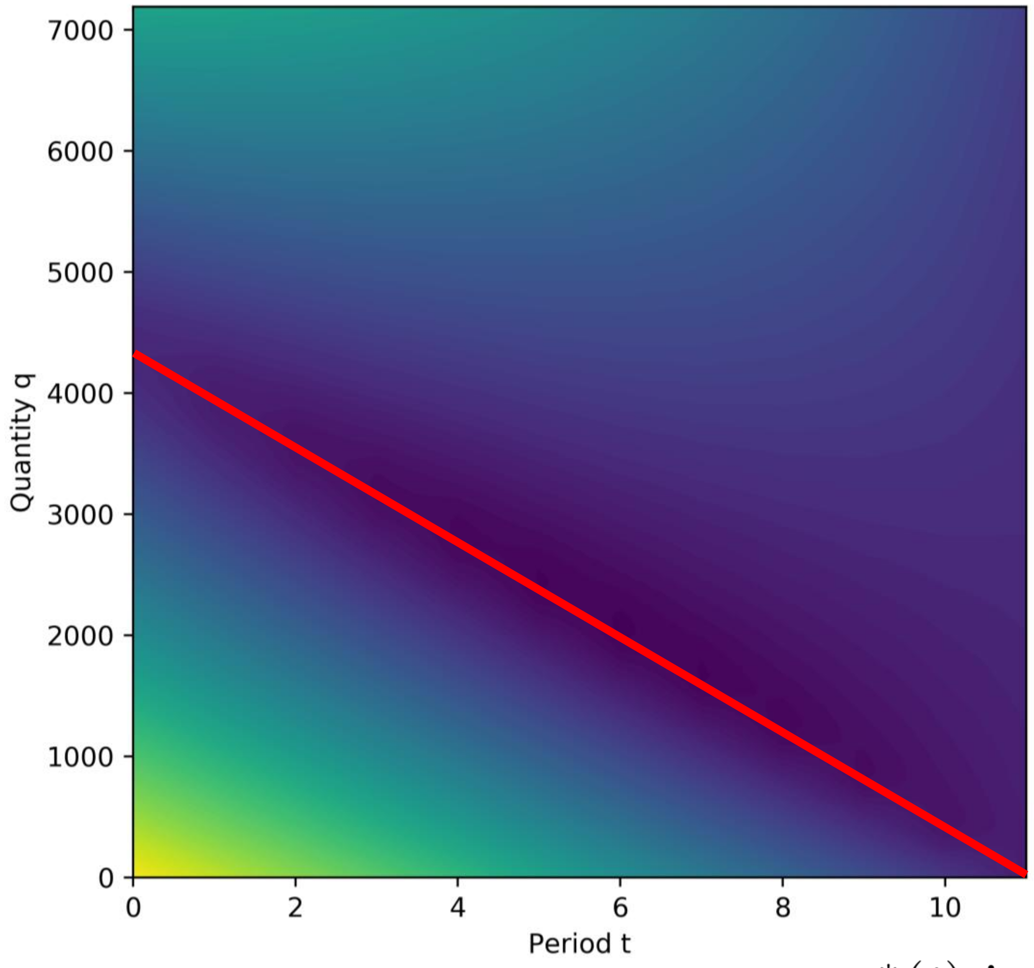

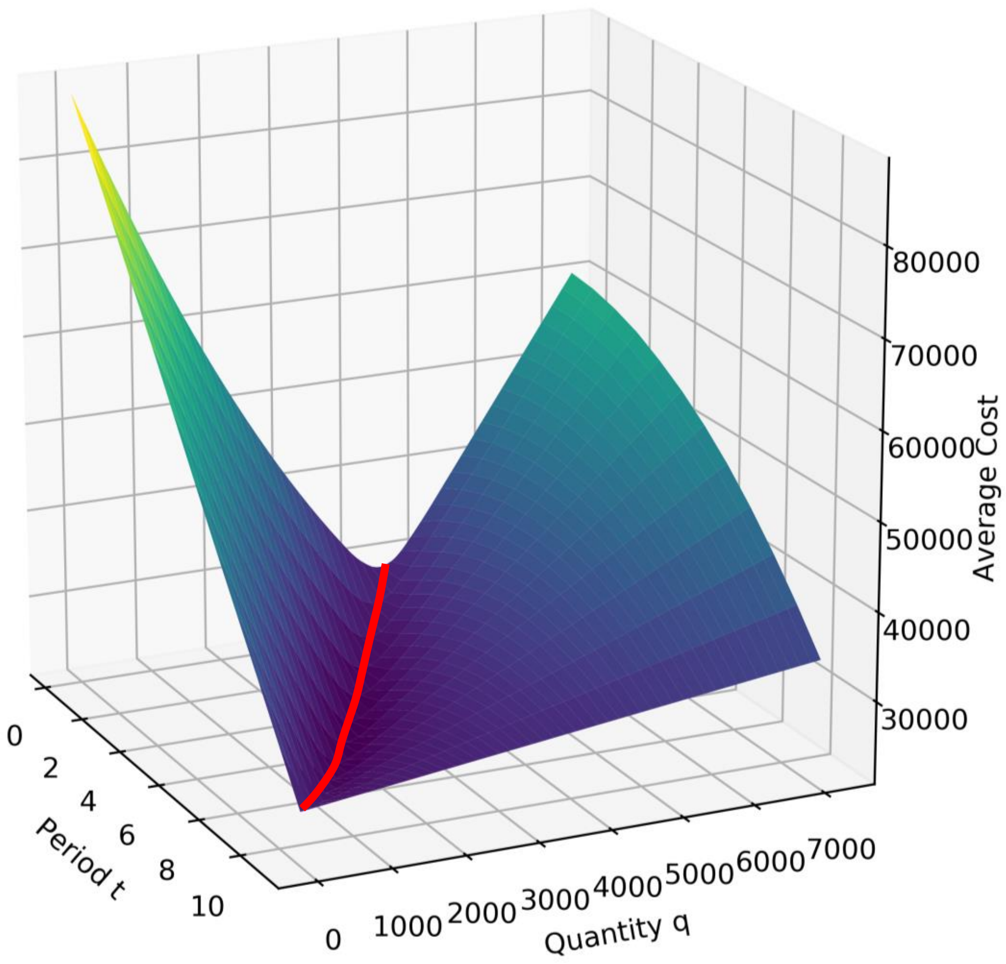

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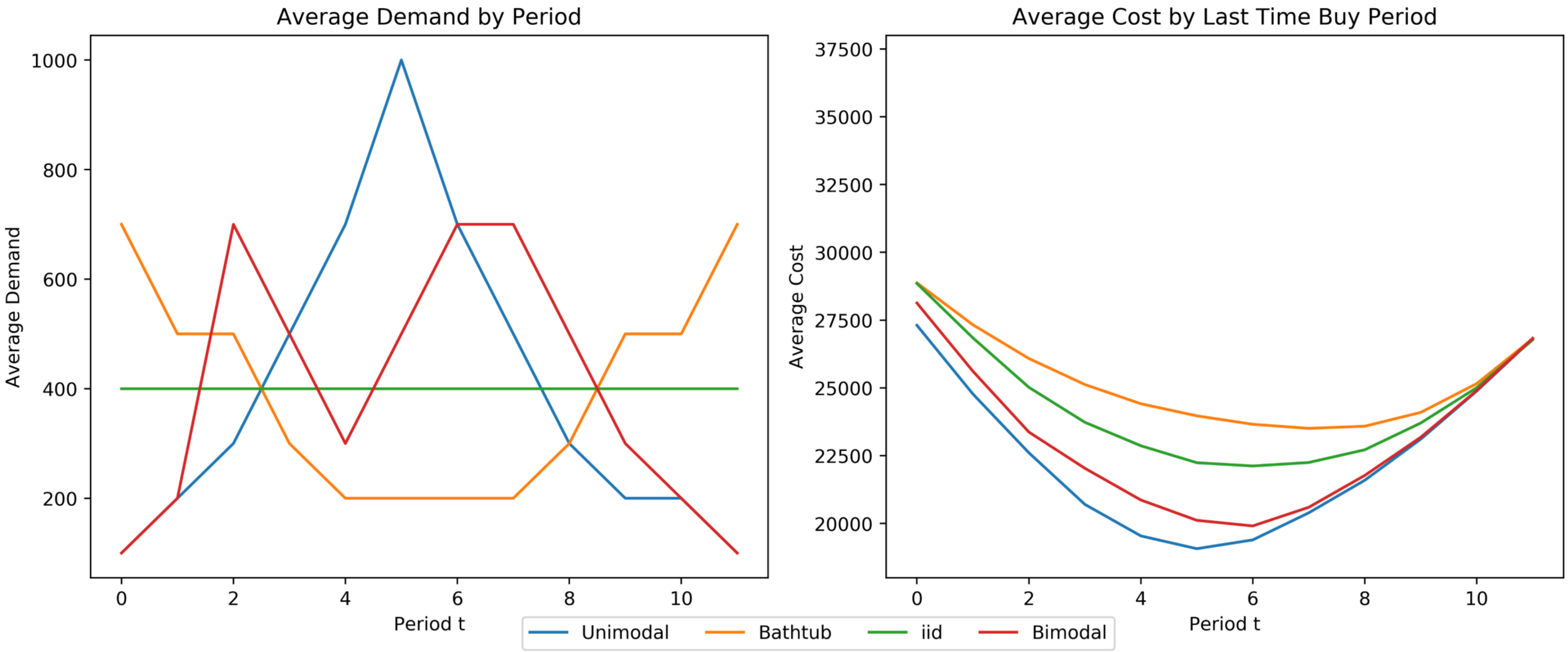

Moral Hazard

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I just sent in my broken golf watch for repair, and the company sent me a brand new golf watch 2.0 instead!

How big of a danger is this moral hazard? It depends on:

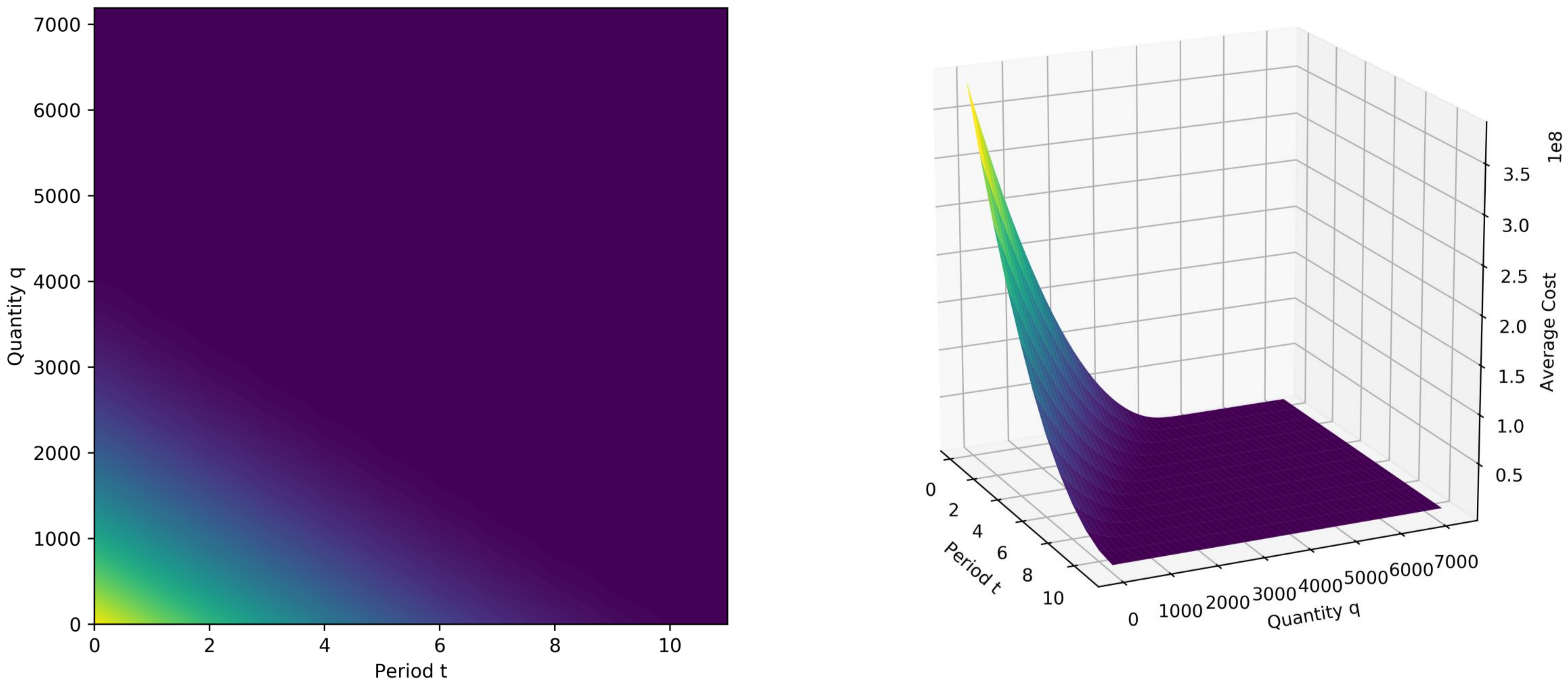
1. The number of new devices given out
2. The time relative to the new product introduction

Quadratic Shortages

$$\min_{t,q} \quad c_f t + c_p \sum_{i=1}^{t} \mathbb{E}[D_i] + c_p q + c_h \sum_{i=t+1}^{T} \mathbb{E}\left[\left(q - \sum_{j=t+1}^{i} D_j\right)^+\right] + c_s \, \mathbb{E}\left[\left(\left(\sum_{i=t+1}^{T} D_i - q\right)^+\right)^2\right]$$

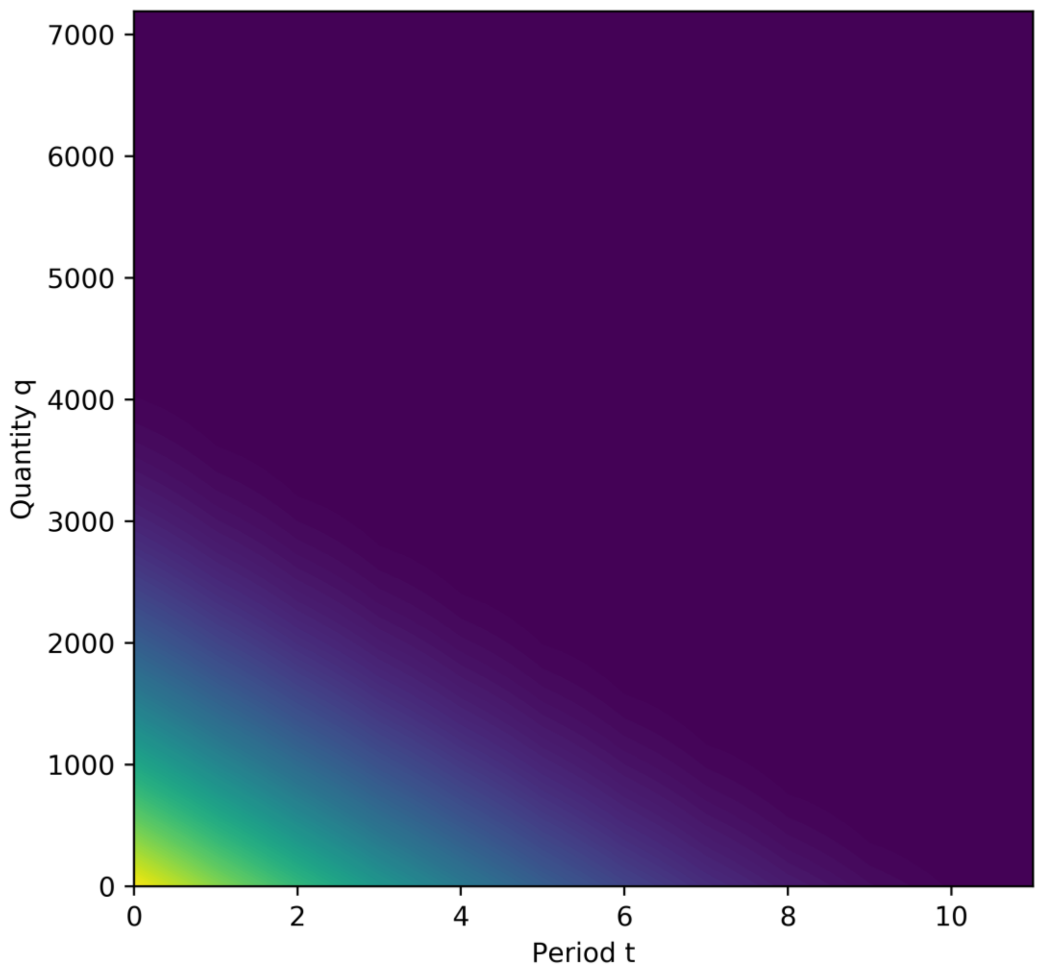
Quadratic Shortages

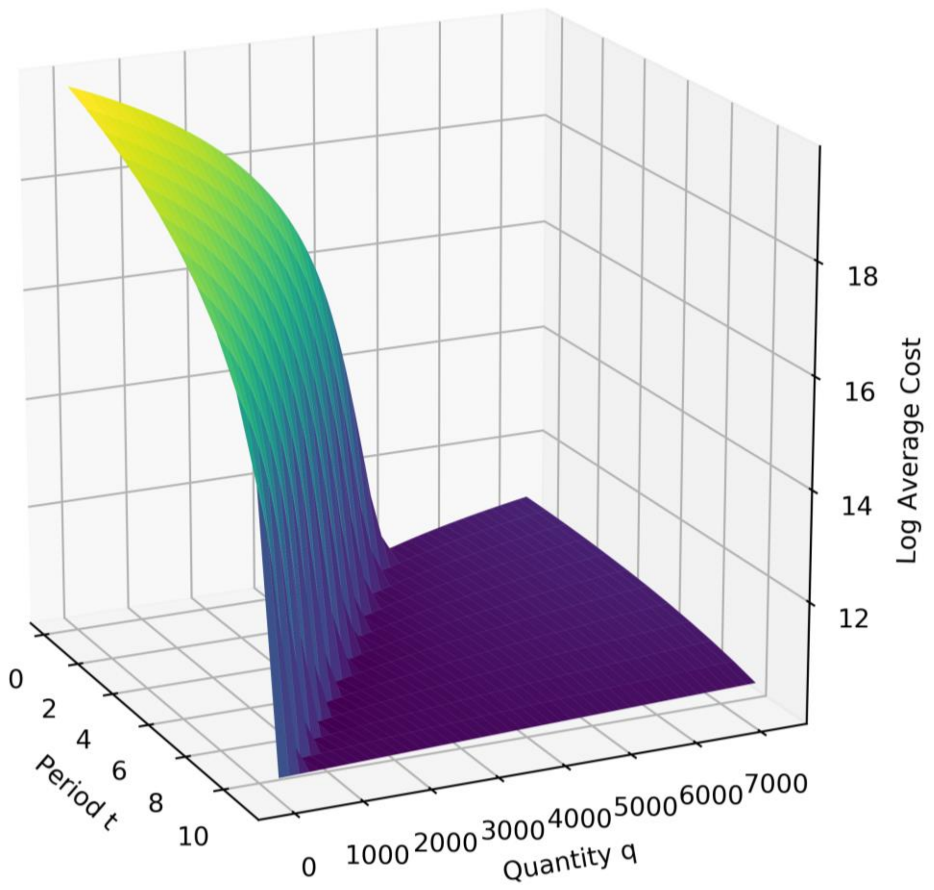
Average Cost for a Variety of Stopping Periods and Order Quantities



Quadratic Shortages

Log of Average Cost for a Variety of Stopping Periods and Order Quantities





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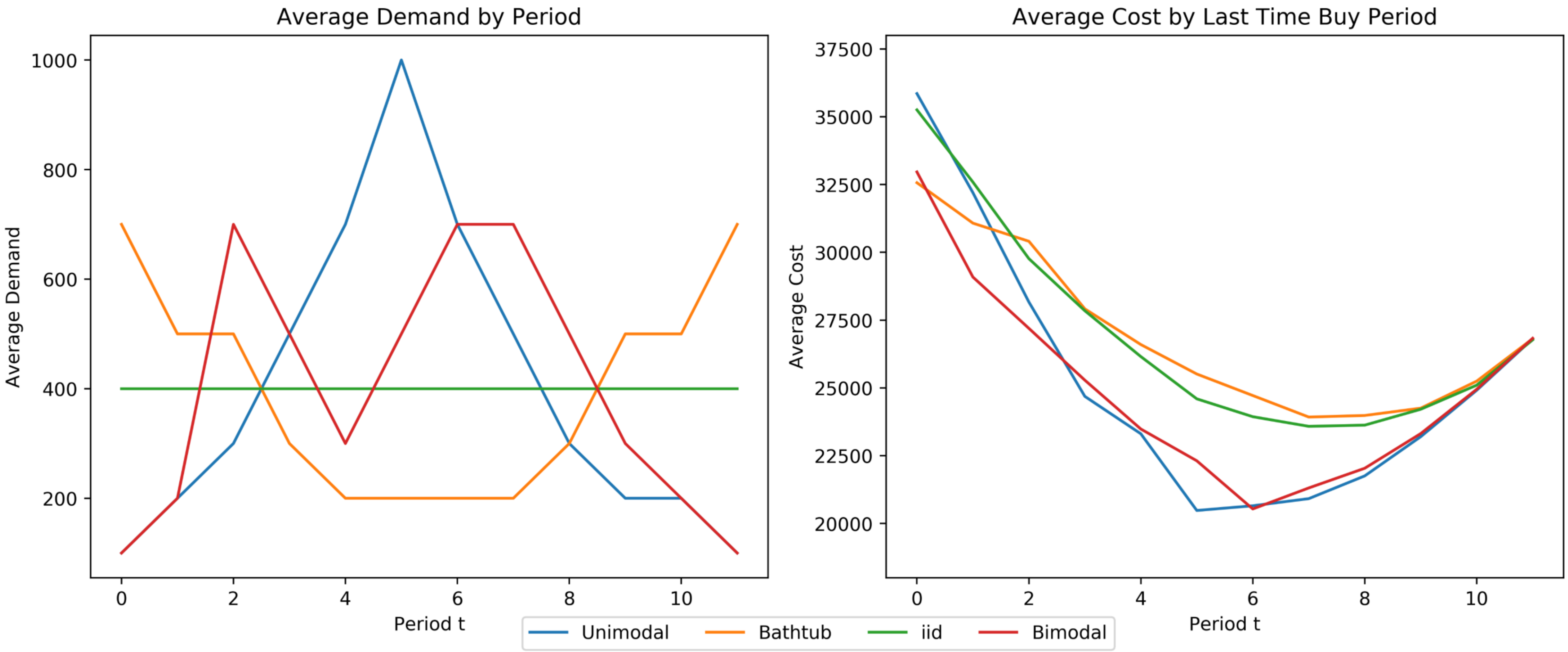

Moral Hazard – Potential Costs

Suppose we used the optimal results from the baseline case with no moral hazard, but then simulated the costs associated in a scenario with moral hazard

Scenario	Immoral Population	Expected Production Cost	Expected Holding Cost	Expected Shortage Cost	Expected Total Cost
Baseline	0	$ 16,906	$ 4,560	$ 500	$ 21,967
Small Moral Hazard – Baseline Solution	5000	$ 16,904	$ 4,518	$ 19,924	$ 41,347
Small Moral Hazard – Optimal Solution	5000	$ 19,167	$ 3,845	$ 385	$ 23,397
Large Moral Hazard – Baseline Solution	20000	$ 16,895	$ 4,487	$ 54,904	$ 76,286
Large Moral Hazard – Optimal Solution	20000	$ 21,162	$ 2,271	$ 60	$ 23,493

Forecasting Warranty Claims

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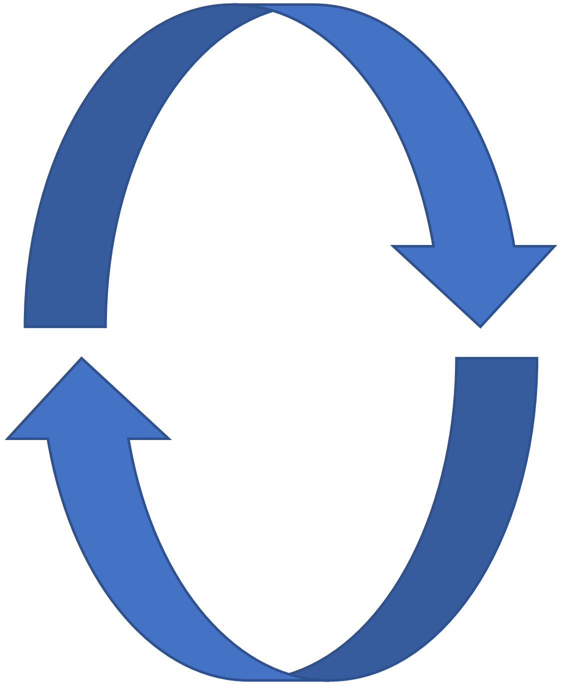
What improvement in forecast is needed to justify a one period delay?

Final Production Period	Expected Cost
Period 6 (optimal)	$21,987
Period 7	$22,203

Difference: $210

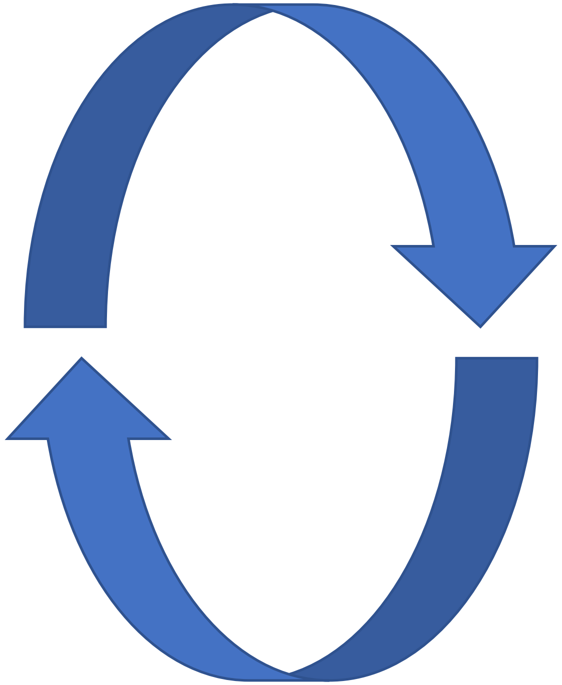
From simulations, we found that reducing the standard deviation from 75 to 65 for periods 7-12 resulted in enough savings to make the delay worthwhile

Forecasting Warranty Claims



Short Product Life Cycles

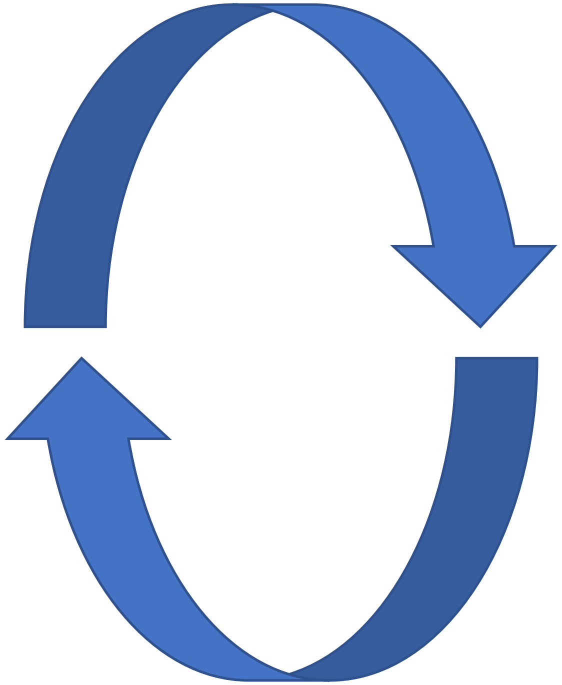
Forecasting Warranty Claims



Short Product Life Cycles

Warranty Expiration

Forecasting Warranty Claims



Short Product Life Cycles

Warranty Expiration

Internet of Things

Thank you